Topological orders in ferroelectrics

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Before we begin / disclaimer

• If you are interested in topology in ferroelectrics, you may want to look at the literature by (at least) the following authors:

Experiment:

- R. Ramesh, L.W. Martin (Berkeley → Rice)
- N. Valanoor (New South Wales)
- D. Muller (Cornell), X. Pan (UC Irvine)
- X.L. Ma (Chinese Ac. Sci. -- Shenyang), …

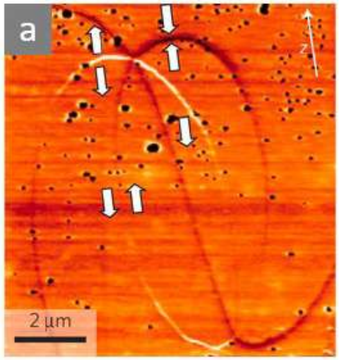
Theory:

- L. Bellaiche, S. Prokhorenko (Arkansas)
- I. Luk'yanchuk (Picardie)
- L.Q. Chen (Penn State)
- J. Junquera (Cantabria), …

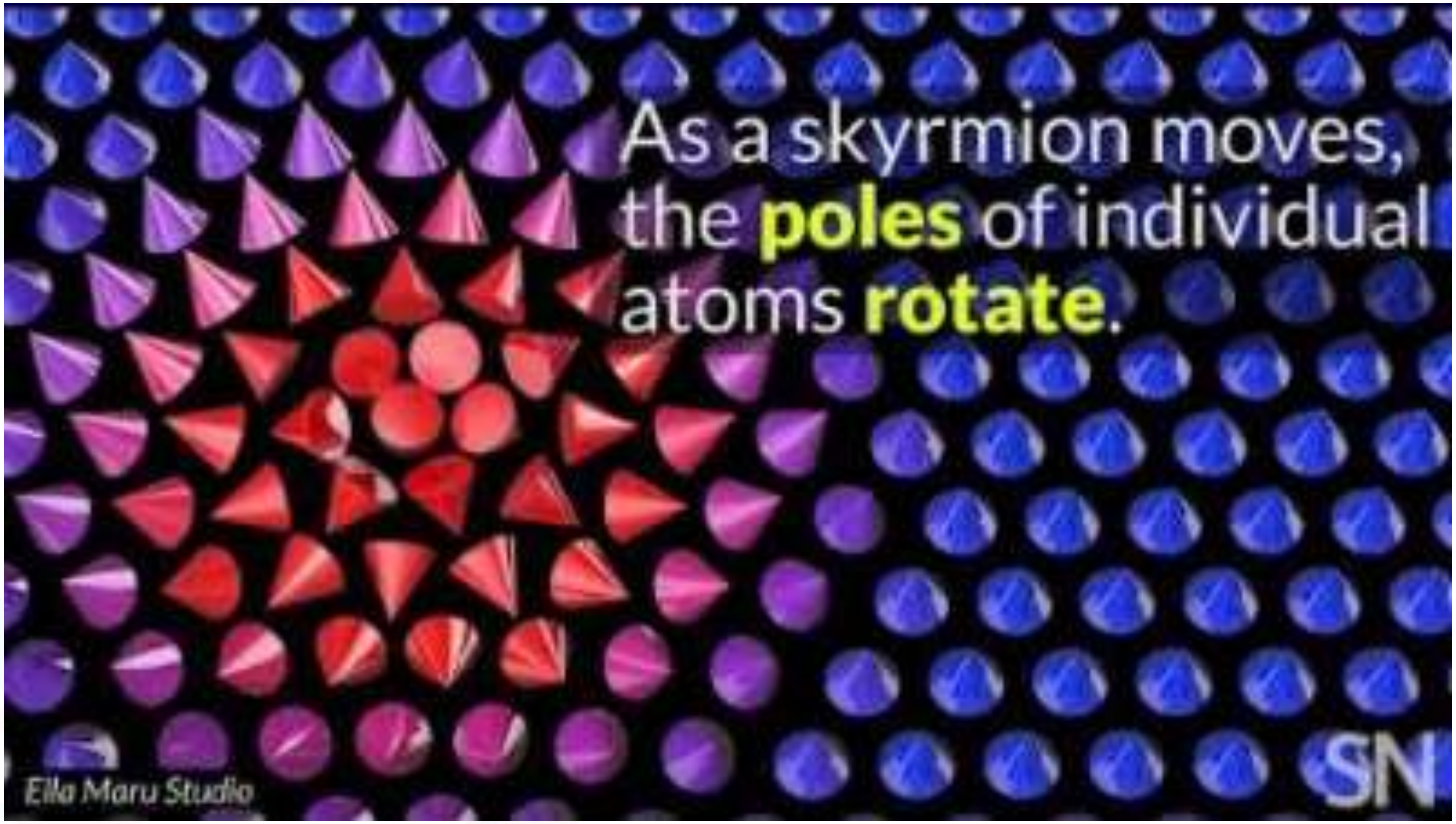
Plus:

- S.-W. Cheong
- M. Fiebig
- D. Meier
- N. Spaldin
- M. Mostovoy
- A. Cano
- …

Meier et al. (2012)

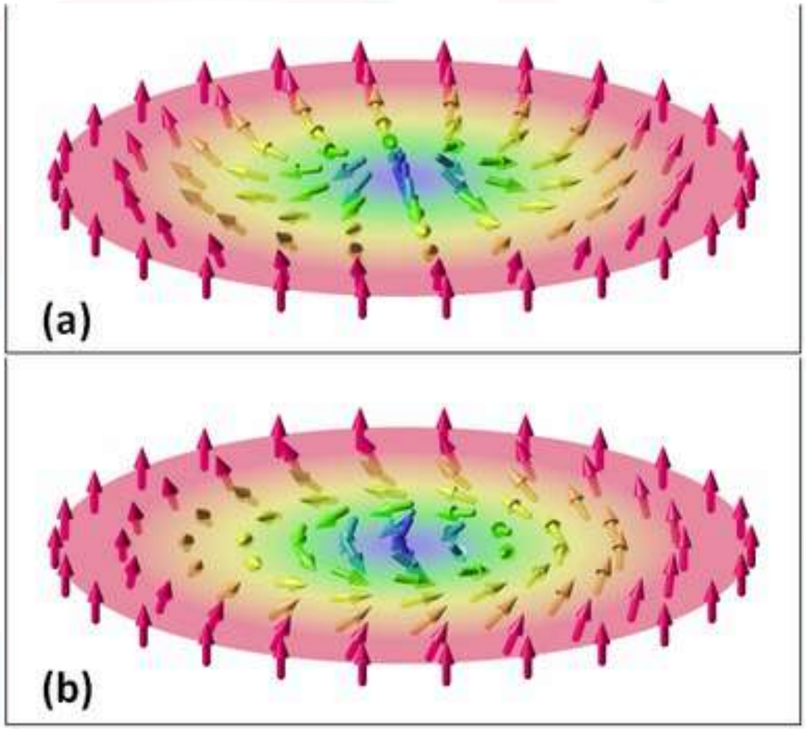


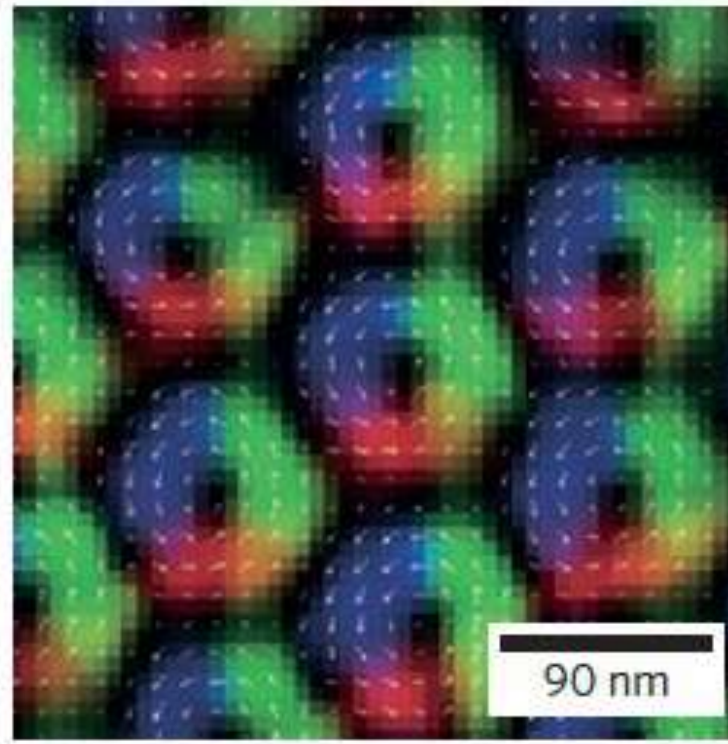
What strikes you about these images?



https://www.youtube.com/@ScienceNewsMag

What strikes you about these images?

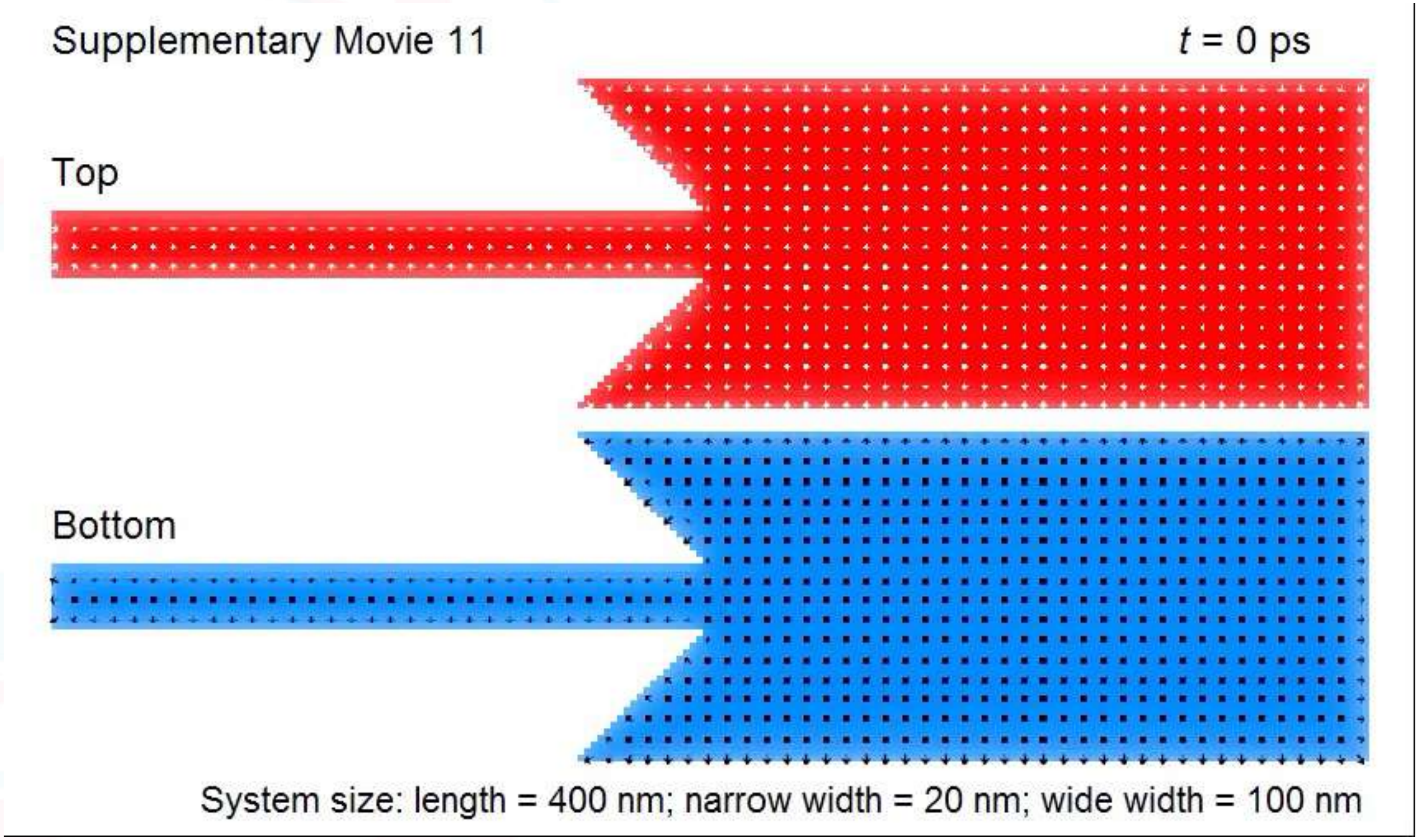




https://en.wikipedia.org/wiki/Magnetic_skyrmion

Yu *et al*., Nature 465, 901 (2010)

What strikes you about these images?

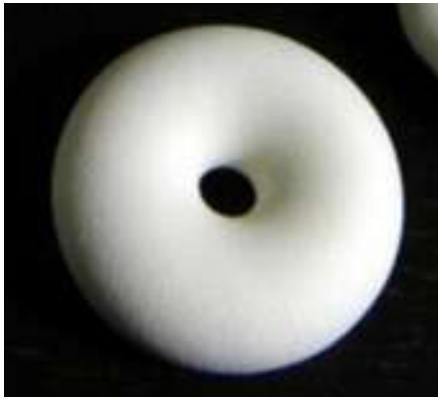


Zhang *et al*., Nature Communications **7**, 10293 (2016)

What about topology?

Image from https://www.shapeways.com/shops/henryseg

What about topology?



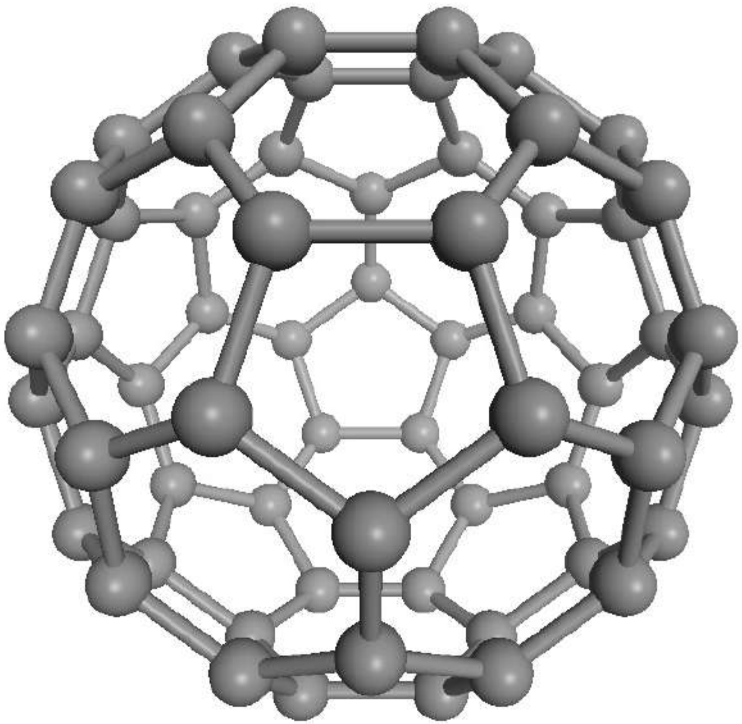
OK, great, they are topologically equivalent. Yet, everything else is pretty much inequivalent between these two.

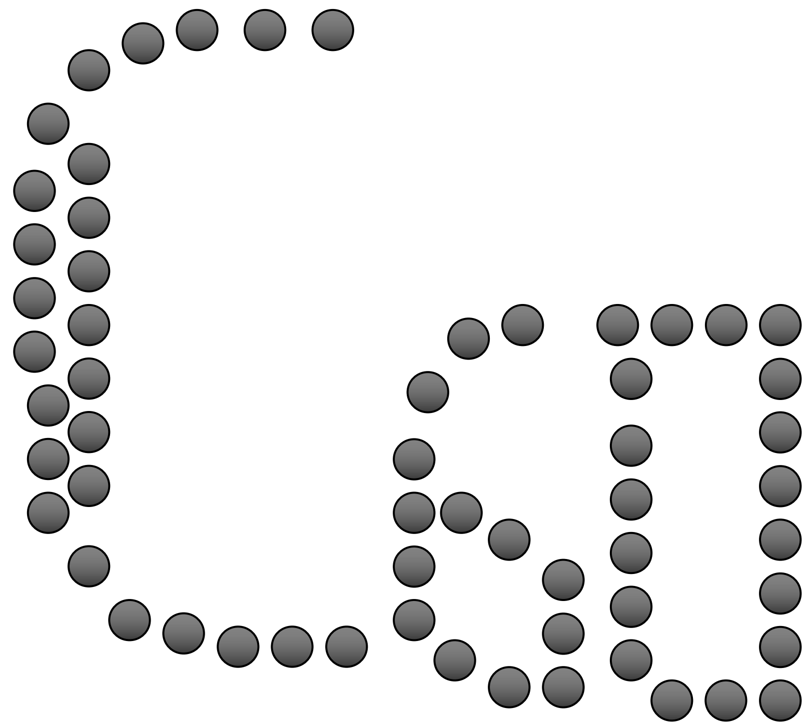
(If you want to have a cup of coffee, which one would you use? :)

Image from https://www.shapeways.com/shops/henryseg

To bring the point home…

What is the difference between these two configurations of 60 carbon atoms?





What's more important, topology or energy?

My personal interest in skyrmions

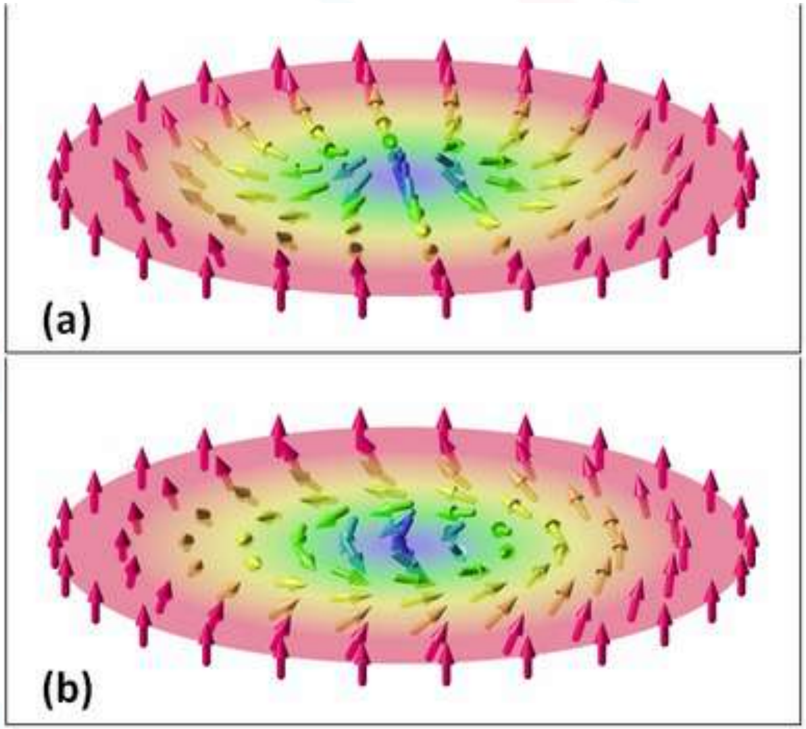
- **<u>Particle-like nature</u>**, diffusive and driven dynamics
- Possibility to create them, erase them, move them, count them, make them interact, …
- Topology will usually be present, but I am not really planning to “use it” (for the moment, at least)

On topology *vs* symmetry:

> At a minimum, homotopy theory provides *the* natural language for the description and classification of defects in a large class of ordered systems. Whether it will eventually gain as wide a currency among condensed matter physicists as, for example, the language and theorems of the theory of group representations, depends both on how large that class of systems proves to be, and on how many of the *nontrivial* topological insights turn out to have direct manifestations in the laboratory.

N.D. Mermin, Rev. Mod. Phys. 51, 591 (1979)

Can we have stable "electric skyrmions"?



Basics of magnetic skyrmions:

• **Non-collinear** structures

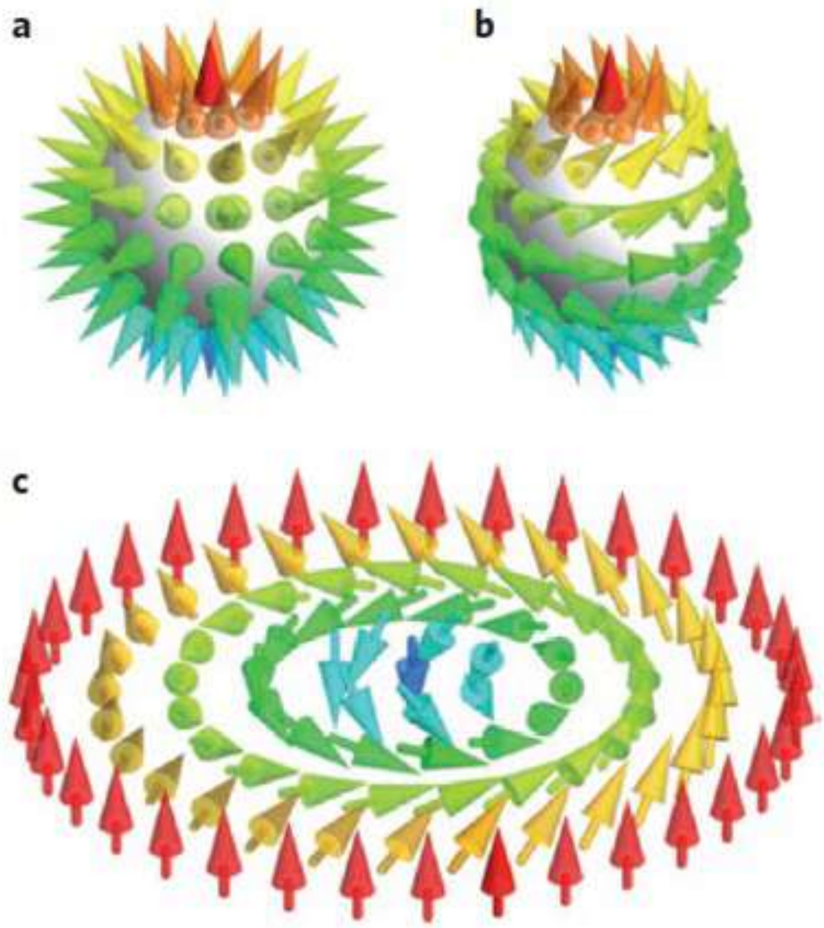
• (Essentially,) Skyrmions display spins in every possible orientation

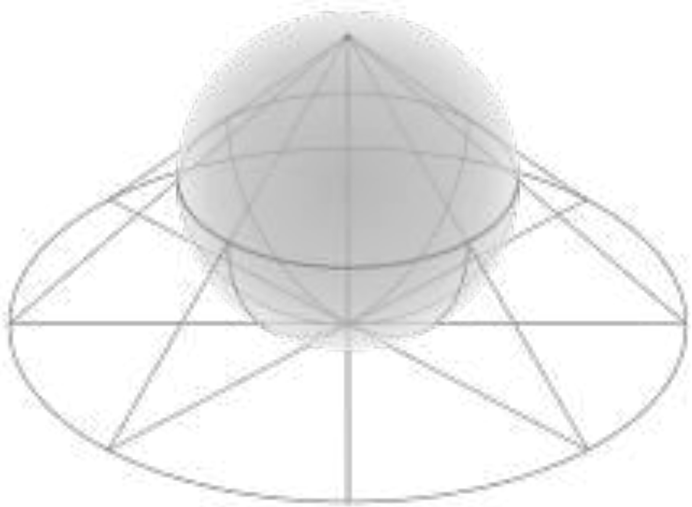
• Source of non-collinearity: Something called **Dzyaloshinskii-Moriya interaction (DMI)**, which occurs in many magnets

• While not the only possible "driving force" for magnetic skyrmions, DMI is a very frequent one and very important historically, so let's stop for a second here.

For more, see e.g. Nagaosa & Tokura, Nature Nanotechnology 8, 899 (2013).

[Every possible orientation…]





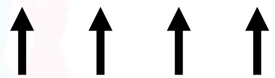
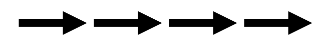
https://en.wikipedia.org/wiki/Stereographic_projection

Pfleiderer, Nat. Phys. 7, 673 (2011)

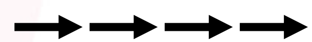
Basic spin Hamiltonian and spin orders

$$H = -\sum_{i\neq j} J\, \vec{S_i}\cdot\vec{S_j} - \sum_{i} K\, S_{i,z}^2 + \sum_{i\neq j} \vec{D}\cdot\vec{S_i}\times\vec{S_j} + \cdots$$

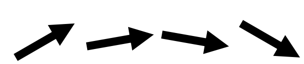
- Symmetric exchange J: favors collinear orders, FM or AFM

E.g., for $J > 0$ we have  or 

- On-site anisotropy K: defines easy magnetic axis or plane

E.g., for $K < 0$ we have 

- **Anti-symmetry exchange $\vec{D}$ (DMI): favors non-collinear spins**

E.g., for $|D| \lesssim |J|$ we have 

[DMI: spin-orbit contribution to exchange, second-order perturbation]

So, is there a DMI for electric dipoles?

TABLE I. Orders of magnitude of the most relevant magnetic and electric interactions in typical magnetic and ferroelectric materials.

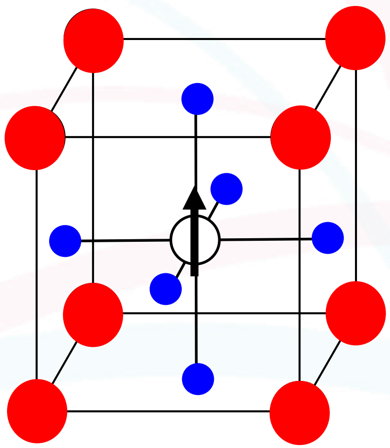
	Magnetic (J)	Electric (J)
Dipolar (at 1 nm)	5×10^{-26}	1×10^{-20}
Short-range	1×10^{-21}	5×10^{-21}
Anisotropy	5×10^{-25}	5×10^{-21}
DMI	5×10^{-22}	5×10^{-22}

- We can have a non-zero “electric DMI”, but it seems relatively small
- On the other hand, the **anisotropy energy** in ferroelectrics is (typically) much greater than in ferromagnets, and greater than the DMI
- Hence, a priori, non-collinear electric dipoles seem unlikely

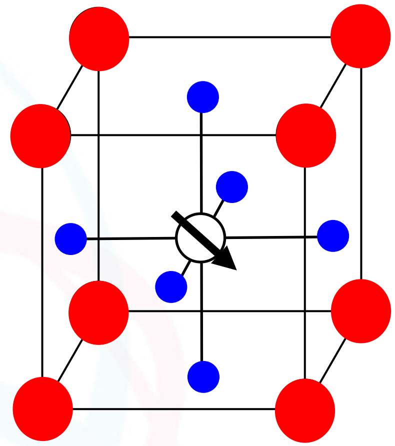
Zhao *et al*., Nature Materials 20, 341 (2021)

Junquera *et al*., Reviews of Modern Physics 95, 025001 (2023)

A word about the anisotropy energy



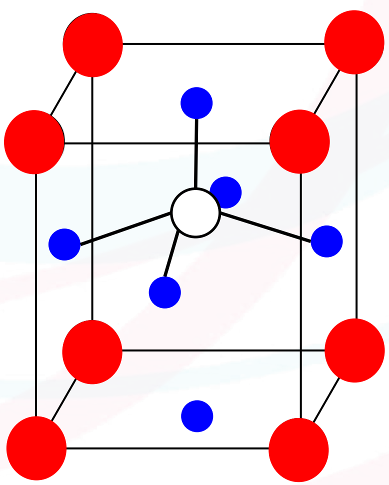
$\vec{S} = S(0,0,1)$



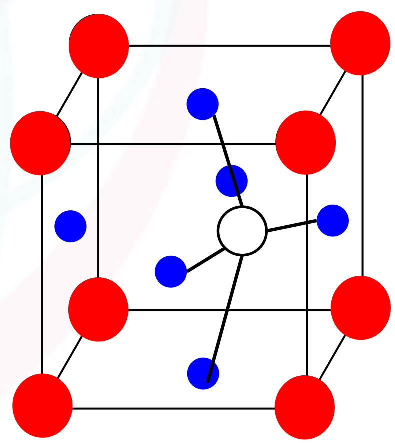
$\vec{S} = S/\sqrt{2}\,(1,1,0)$

On-site magnetic anisotropy comes from the spin-orbit interaction

→ relatively small



$\vec{P} = (0,0,P)$



$\vec{P} = (P', P', 0)$

Polarization anisotropy comes from changes in chemical bonds & cell deformations

→ relatively large

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- There is indeed a non-zero “electric DMI”, but it seems relatively small
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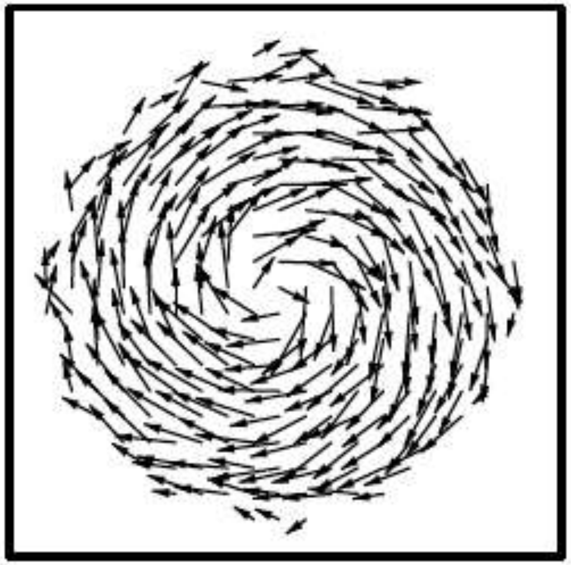
- **Hence, a priori, non-collinear electric dipoles seem unlikely**

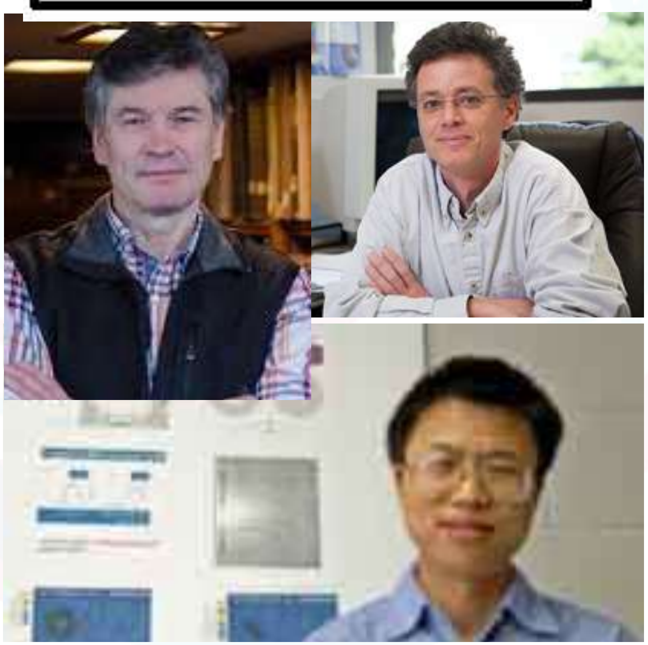
Zhao *et al*., Nature Materials 20, 341 (2021)

Junquera *et al*., Reviews of Modern Physics 95, 025001 (2023)

And yet…

Naumov, Bellaiche & Fu, Nature 432, 737 (2004)

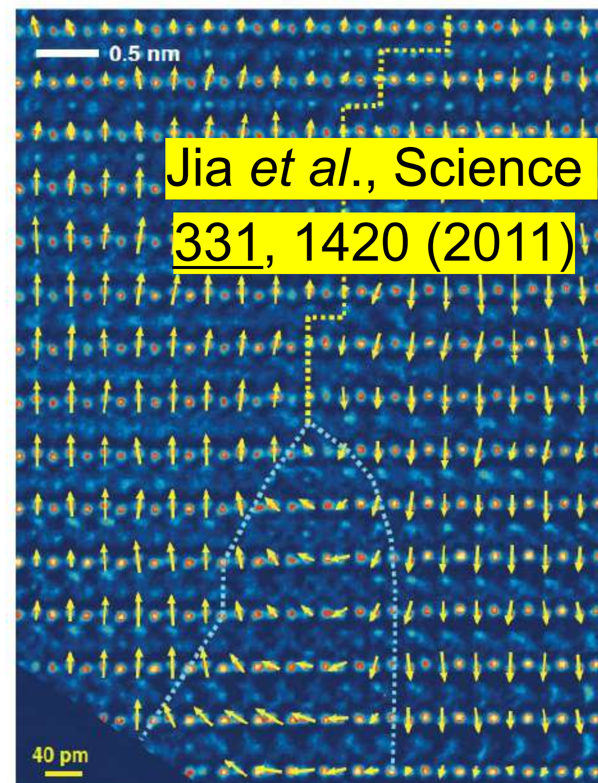




In 2004: Naumov *et al*. predicted **dipole vortexes** in ferroelectric nanorods !

In 2008: Aguado-Puente and Junquera (PRL 100, 177601) predicted **closure domains** in ferroelectric ultrathin films

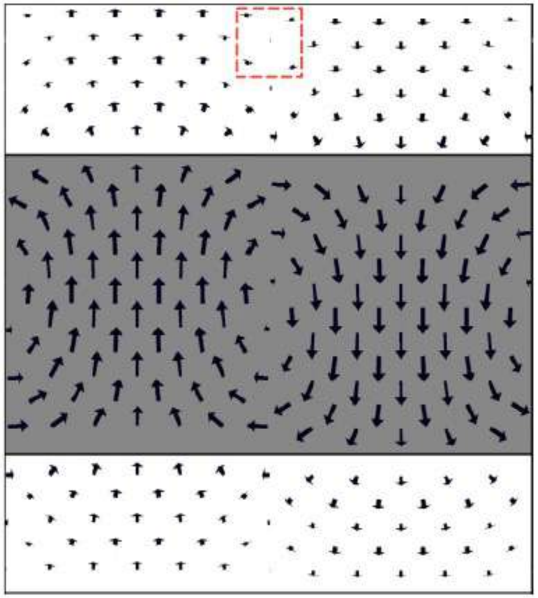
In 2011: Related experimental evidence from STEM !



Jia *et al*., Science 331, 1420 (2011)

Then, $PbTiO_3/SrTiO_3$ entered the picture

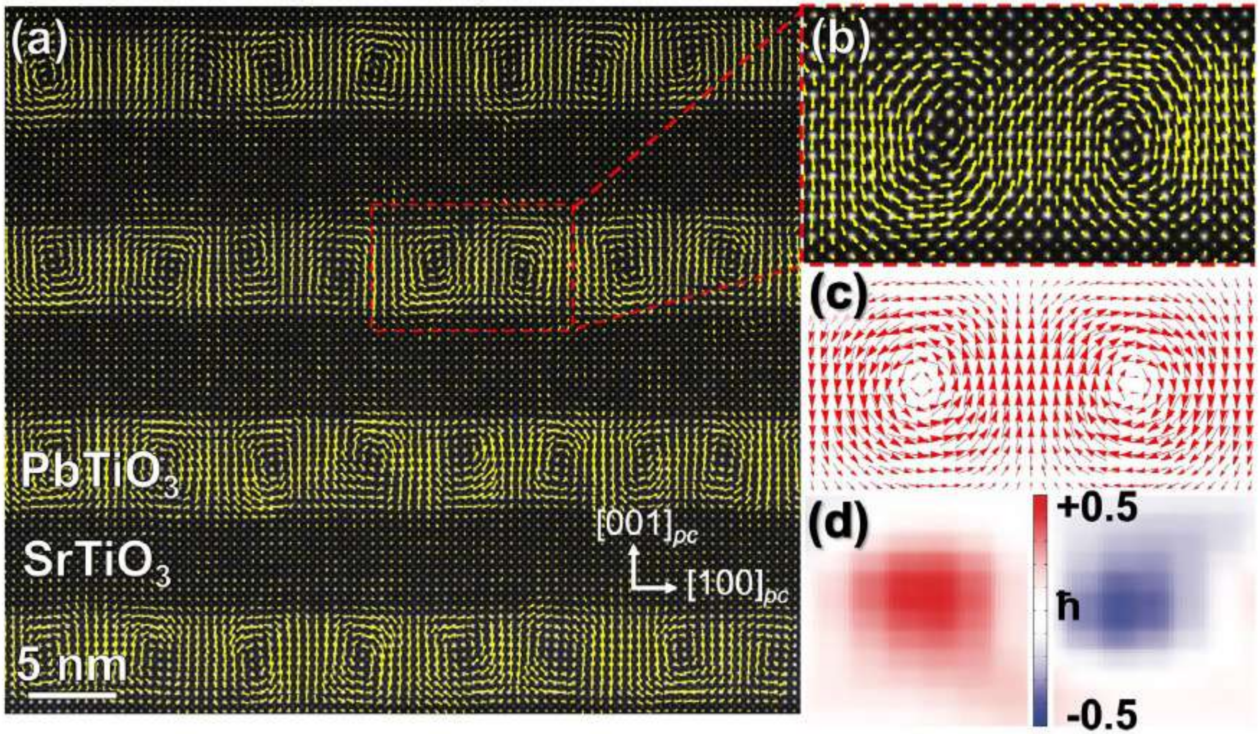
DFT prediction (2012)



Aguado-Puente & Junquera, PRB 85, 184105 (2012)

consistent with experiments by Zubko *et al*. (e.g., Phys. Rev. Lett. 104, 187601 (2010))

Very famous experimental demonstration (2016)



Yadav *et al*., Nautre 530, 198 (2016)

So, how can we have non-collinear order after all…?

TABLE I. Orders of magnitude of the most relevant magnetic and electric interactions in typical magnetic and ferroelectric materials.

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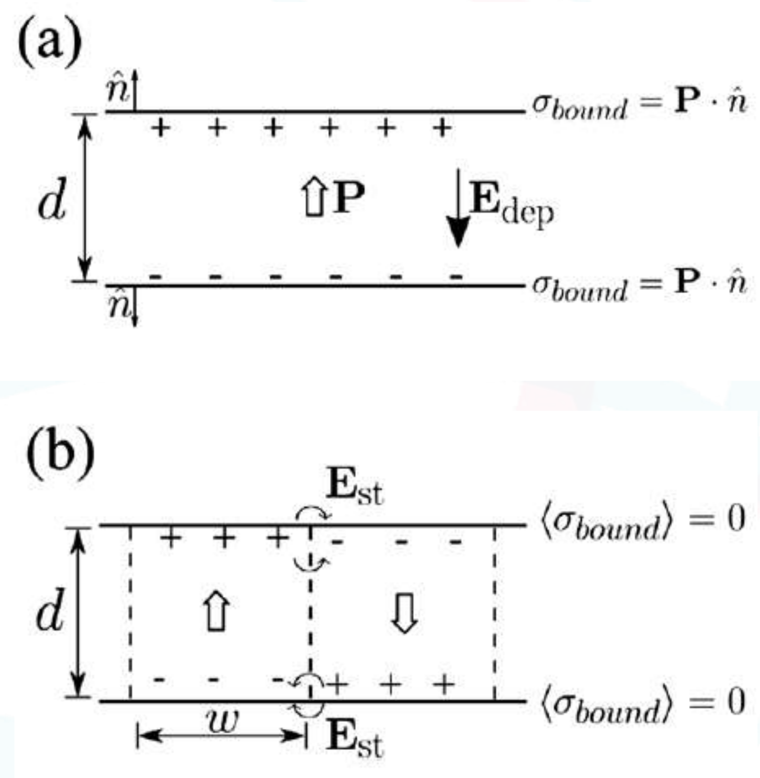
DMI	5×10^{-22}	5×10^{-22}

- Electrostatic dipole-dipole interactions dominate
- Everything else ”adapts” in order to minimize electrostatic energy

- Note: By contrast, “magnetostatic” couplings are very weak. They play no significant role as regards skyrmion stabilization (usually, at least).

Junquera *et al*., Reviews of Modern Physics 95, 025001 (2023)

Electrostatics in ferroelectrics, well known!



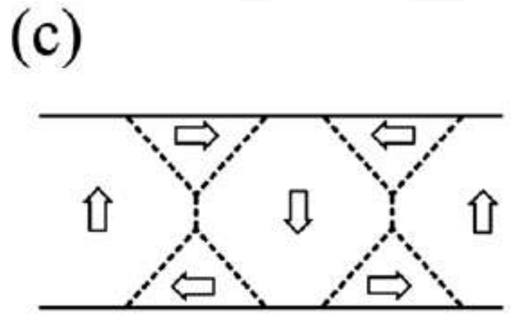
The "depolarizing field" ($\mathrm{E}_{\mathrm{dep}}$) will typically "kill" the homogeneous polarization

However, $\mathrm{E}_{\mathrm{dep}}$ can be drastically reduced if the ferroelectric breaks into domains, so that only small "stray fields" (E_{st}) remain

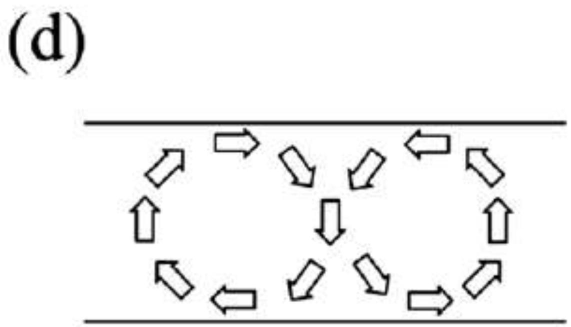
[Where the thickness of the domains (w) depends on the thickness of the layer (d) according to Kittel's law (Physical Review 70, 965 (1946))]

Junquera *et al*., Reviews of Modern Physics 95, 025001 (2023)

What about ultrathin ferroelectric films?



The stray fields can be further reduced by forming flux-closure domains

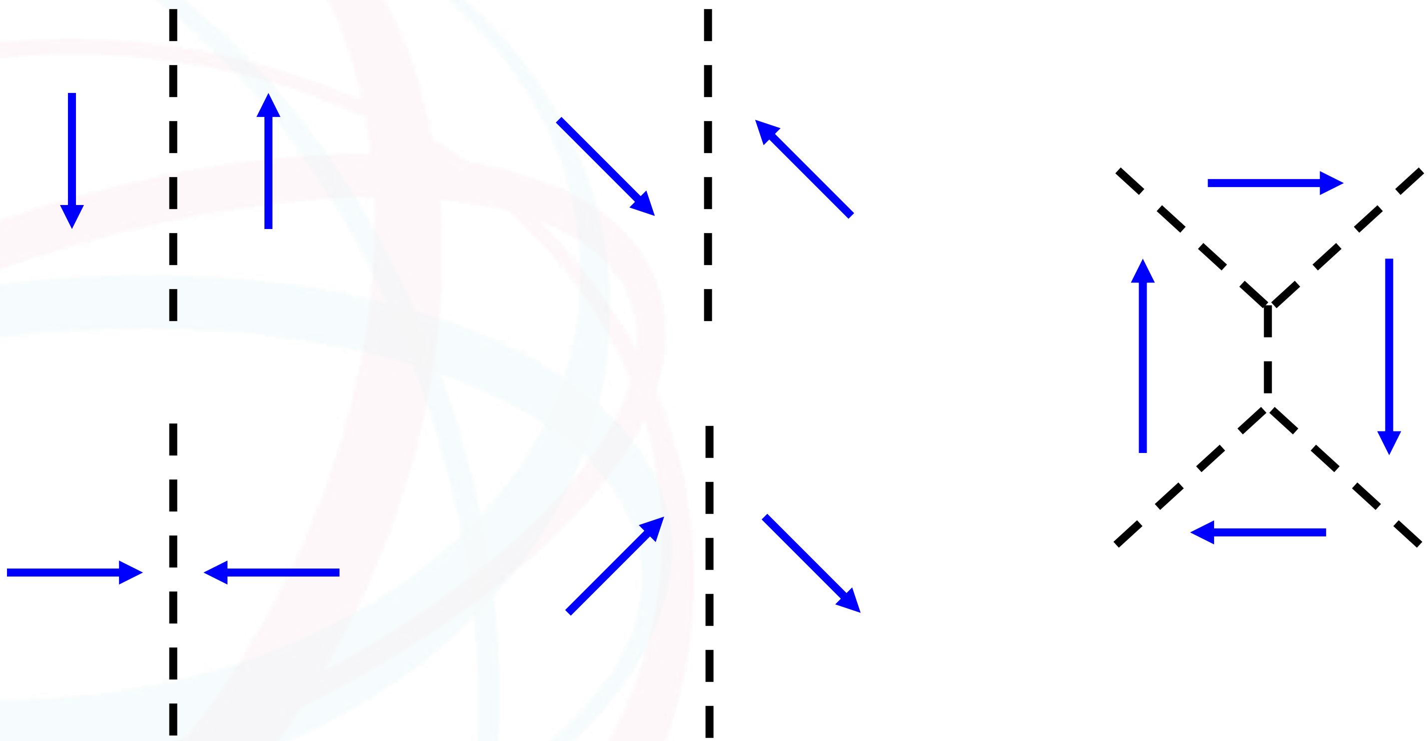


If the thickness of the layer is very small, we can even obtain dipole vortexes !

No polarization at the center of the domain wall / vortex !!

Junquera *et al*., Reviews of Modern Physics 95, 025001 (2023)

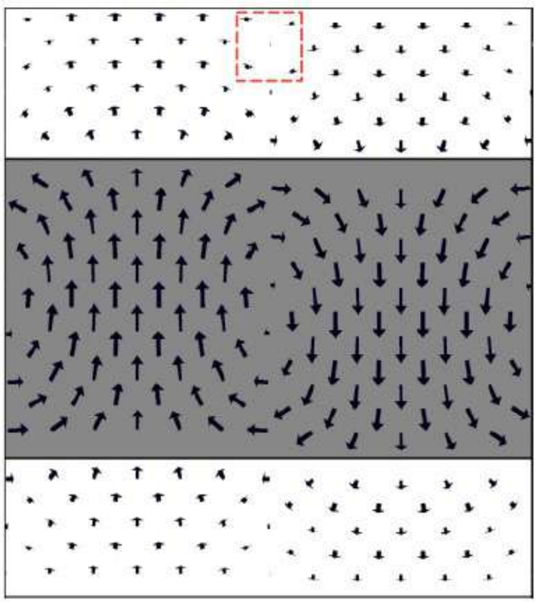
[A word about these domains and domain walls]

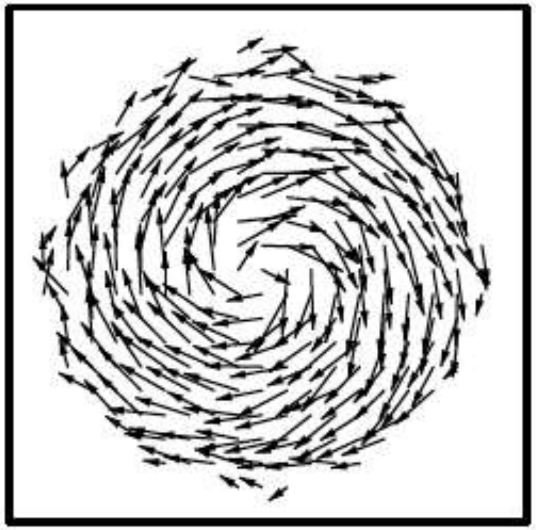


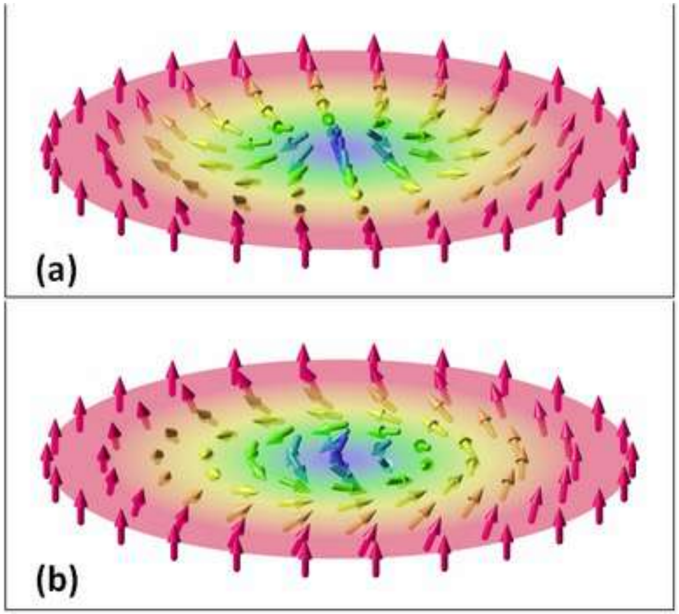
$$\rho_{bound} = -\nabla \cdot \vec{P}$$

This is great, but…

… is there a skyrmion here?







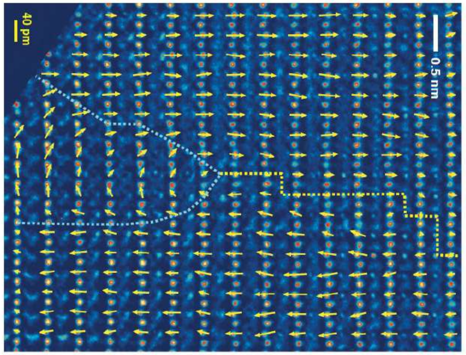
(If not, what are we missing?)

Theory → Experiment

Non-collinear electric dipoles

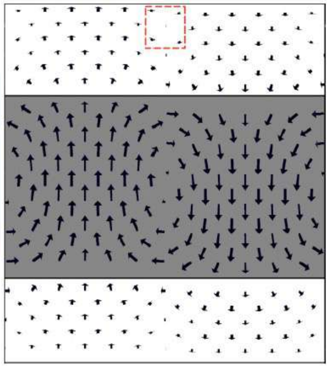
2004

2011

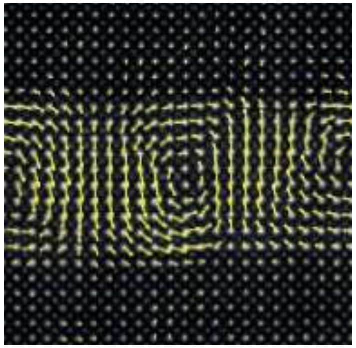


Vortex-like domain walls

2012



2016



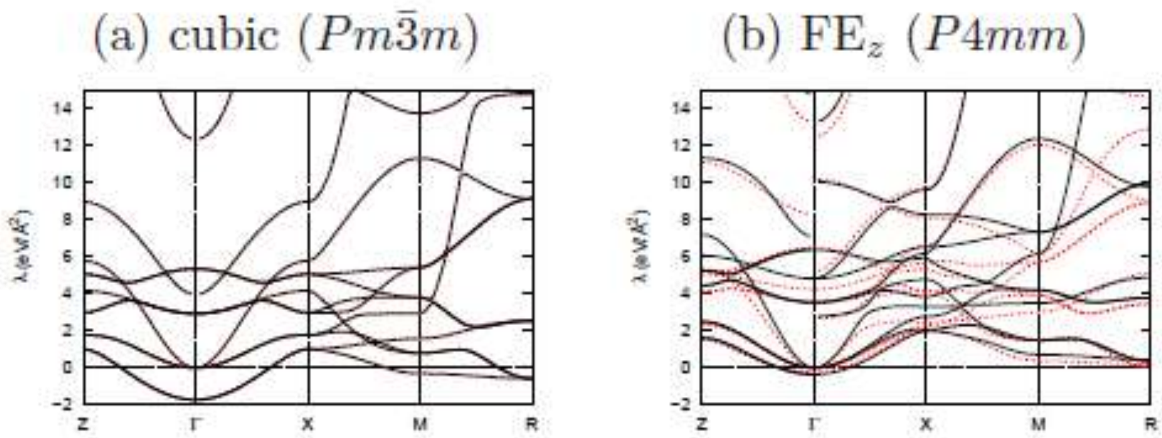
Electric skyrmion

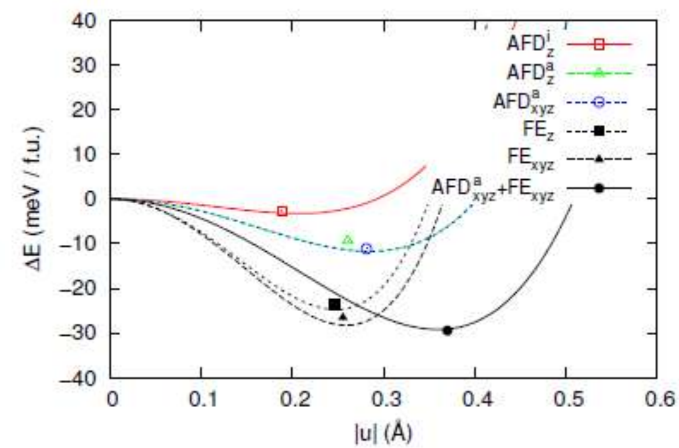
?

?

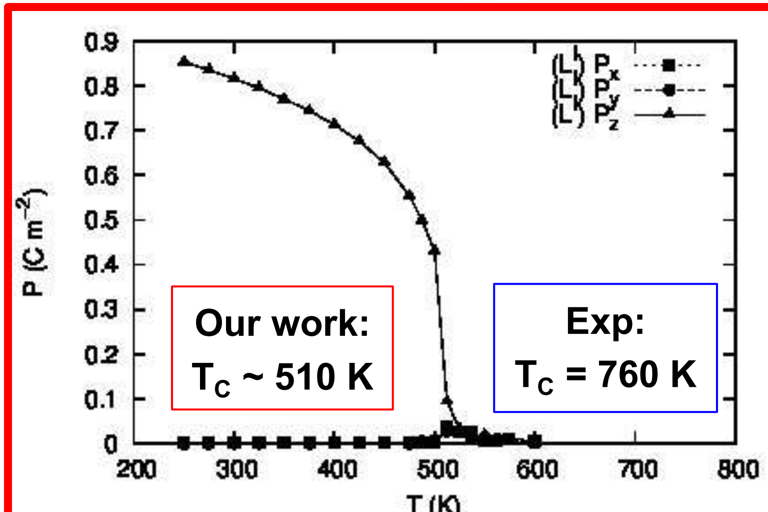
Questions?

Second principles model for $PbTiO_3$ (and $SrTiO_3$)





Structure	Method	u^{Γ}_{Pbz}	u^{Γ}_{Tiz}	u^{Γ}_{O1z}	u^{Γ}_{O3z}	O_6 rot.	Energy
FE_z ($P4mm$)	LDA	0.179	0.072	−0.104	−0.043	—	−23.7
	Model	0.180	0.073	−0.105	−0.043	—	−24.8
FE_{xyz} ($R3m$)	LDA	0.104	0.048	−0.063	−0.027	—	−26.6
	Model	0.105	0.049	−0.063	−0.028	—	−28.3
AFD^a_z ($I4/mcm$)	LDA	—	—	—	—	5.4	−9.4
	Model	—	—	—	—	5.9	−11.7
AFD^a_{xyz} ($R\bar{3}c$)	LDA	—	—	—	—	3.4	−11.2
	Model	—	—	—	—	3.4	−11.7
AFD^i_z ($P4/mbm$)	LDA	—	—	—	—	3.9	−2.7
	Model	—	—	—	—	4.3	−3.3
FE_{xyz}+AFD^a_{xyz} ($R3c$)	LDA	0.096	0.047	−0.058	−0.026	2.8	−29.5
	Model	0.098	0.047	−0.060	−0.026	2.1	−29.5

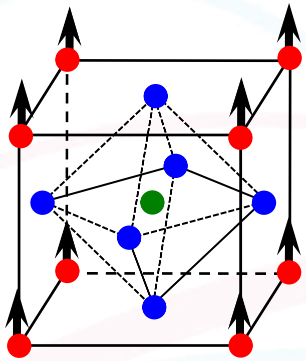


Displacements: Angstrom ; Rotations: degrees ; Energies: meV/f.u.

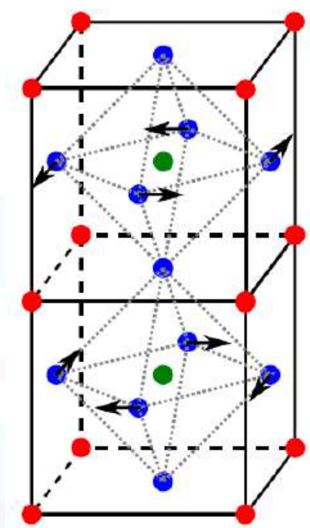
Excellent accuracy reproducing first-principles data

Wojdeł, Hermet, Ljungberg, Ghosez & Íñiguez, JPCM 25, 305401 (2013)

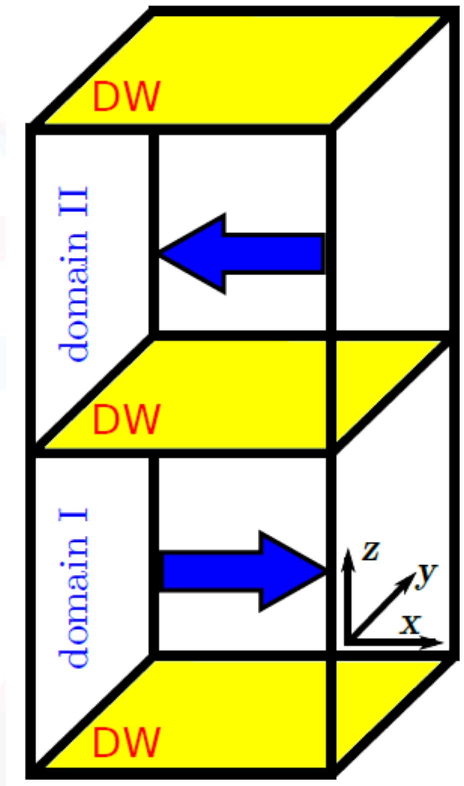
Ferroelectric walls, expectations circa 2010

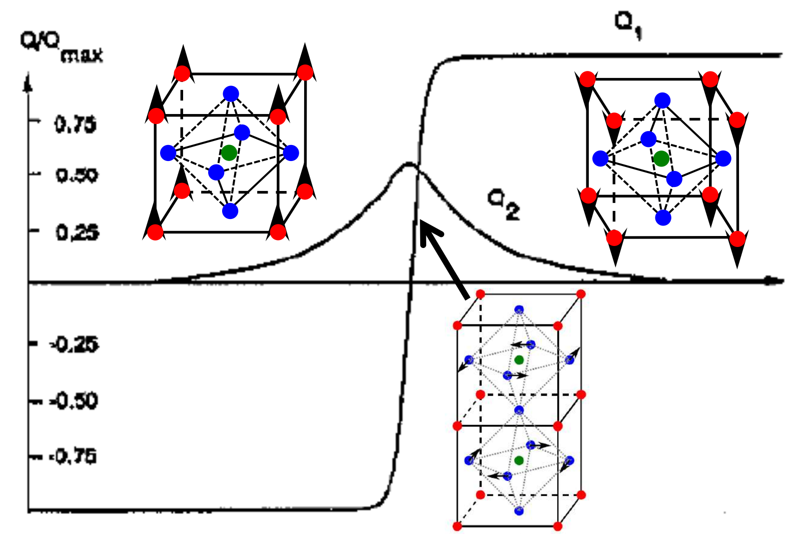


Ferroelectric polarization



Rotations of the O_6 octahedra

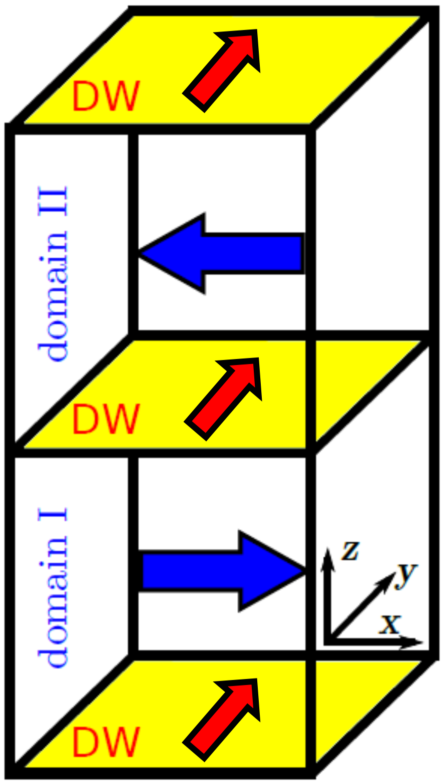


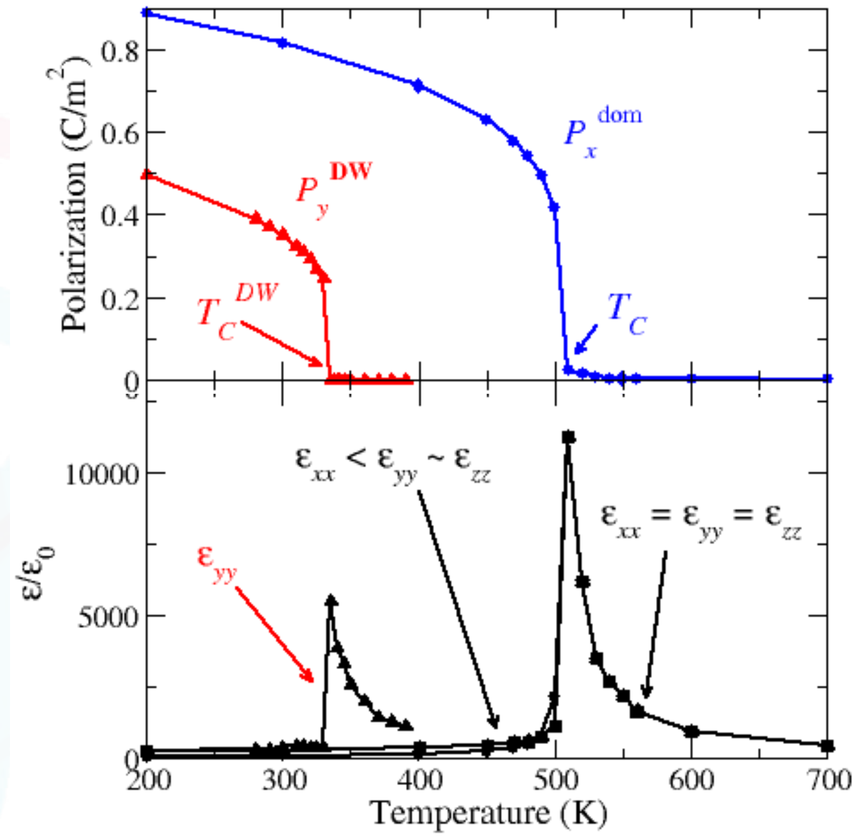


$$G = \tfrac{1}{2}\gamma_1(\nabla Q_1)^2 + \tfrac{1}{2}\gamma_2(\nabla Q_2)^2 + \tfrac{1}{2}A_1Q_1^2 + \tfrac{1}{4}B_1Q_1^4 + \tfrac{1}{2}A_2Q_2^2 + \tfrac{1}{4}B_2Q_2^4 + \lambda Q_1^2Q_2^2.$$

- Houchmandzadeh, Lajzerowicz & Salje, JPCM 3, 5163 (1991).
- Lajzerovicz & Niez, J. Phys. Lett. 40, L165 (1979).
- Tagantsev, Cross & Fousek, *Domains in Ferroic Crystals and Thin Films* (Springer, 2010).
- Taherinejad *et al.*, Phys. Rev. B 86, 155138 (2012).
- Marton, Stepkova & Hlinka, Phase Transit. 86, 103 (2013).

Ferroelectricity at ferroelectric domain walls





DWs:

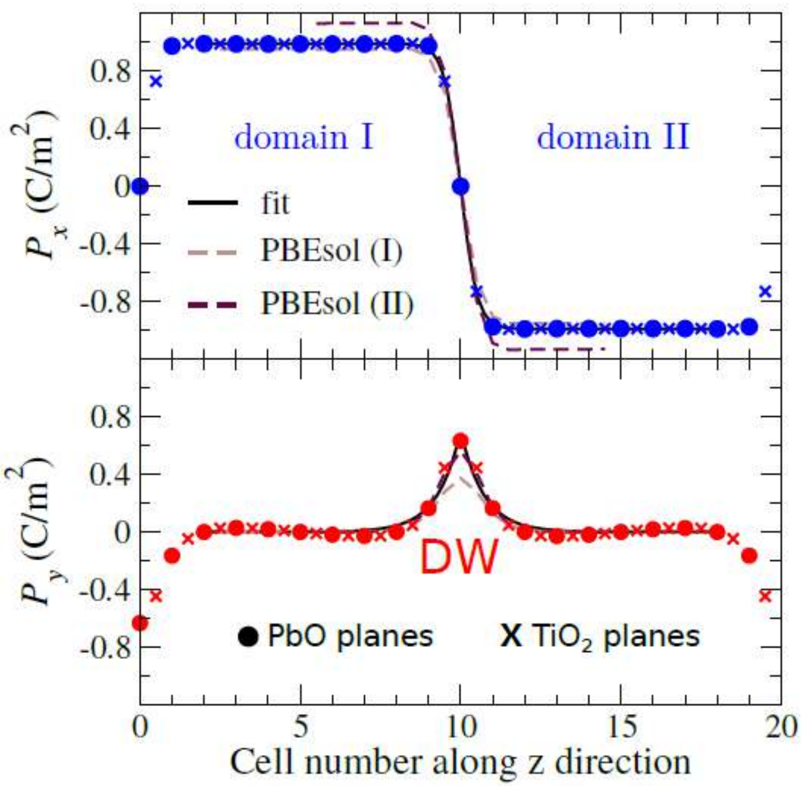
Ising

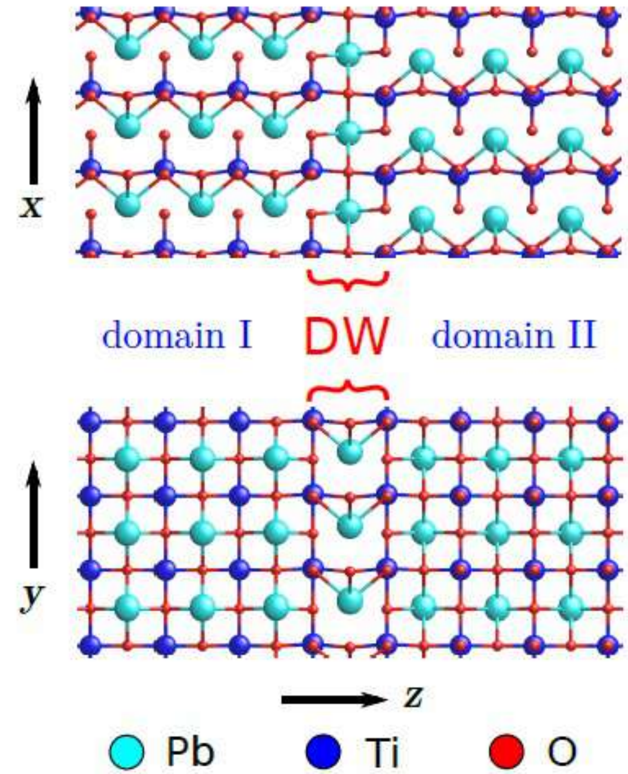
↓

Bloch

Wojdeł & Íñiguez, Phys. Rev. Lett. 112, 247603 (2014)

Bloch DW, explicitly confirmed from DFT



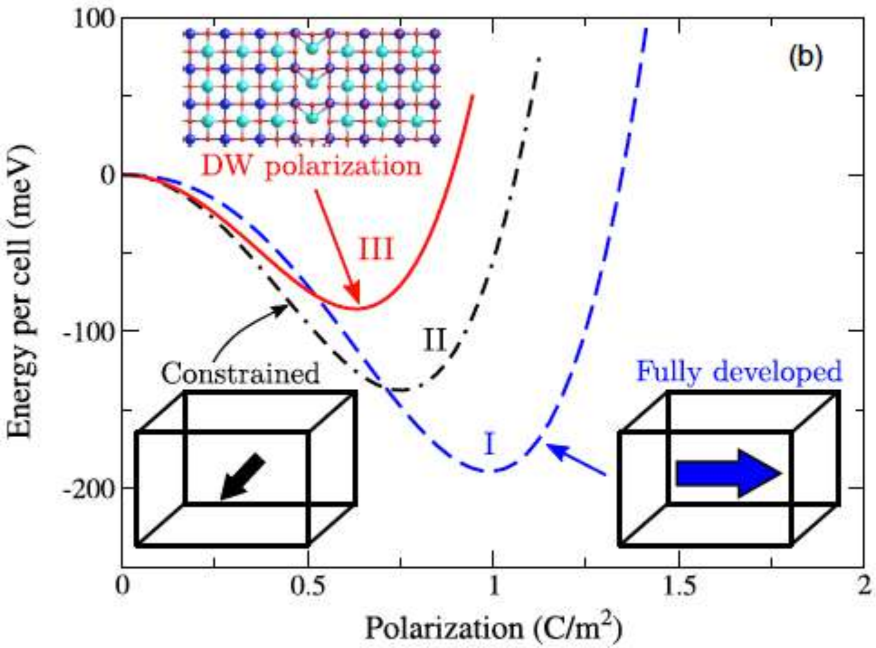


- Confirmed independently by other authors via DFT studies

  → Wang *et al*., JAP 116, 224105 (2014); Liu & Cohen, JPCM 29, 244003 (2017)

- Very similar to behavior predicted for DWs in $BaTiO_3$ [DFT, Phys. Rev. B 86, 155138 (2012)] and $SrTiO_3$ [phenomenology, PRB 64, 224107 (2001)]

Why a DW polarization



(I) Polarization well in bulk $PbTiO_3$

(II) Bulk-like polarization with unfavorable c/a (as at DW)

(III) Polarization well at DW

In spite of unfavorable constraints (strain, dimensionality) PTO's ferroelectric instability active at DW

- Phenomenological interpretation

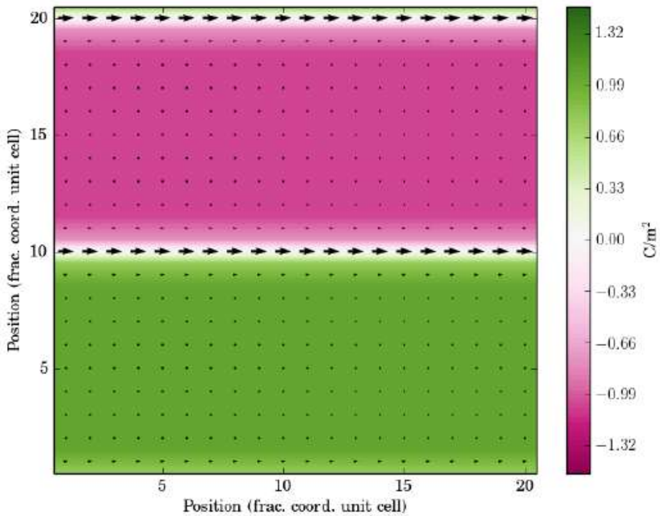
$$G = \tfrac{1}{2}\gamma_1(\nabla Q_1)^2 + \tfrac{1}{2}\gamma_2(\nabla Q_2)^2 + \tfrac{1}{2}A_1Q_1^2 + \tfrac{1}{4}B_1Q_1^4 + \tfrac{1}{2}A_2Q_2^2 + \tfrac{1}{4}B_2Q_2^4 + \lambda Q_1^2Q_2^2.$$

- We usually think of Q_1 and Q_2 are two different distortions.
- Here we have equivalent $Q_1 = P_z$ and $Q_2 = P_y$; they compete ($\lambda > 0$)
- Once Q_1 condenses to form domains, Q_2 effectively becomes the "weaker" order and occurs only at the walls

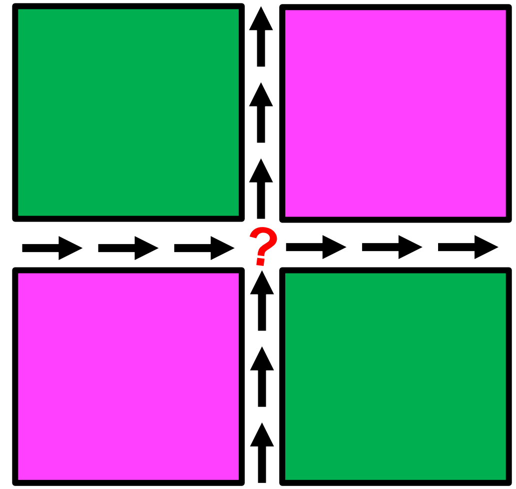
You can't always get what you want…

Ideal planar 180° DW in $PbTiO_3$

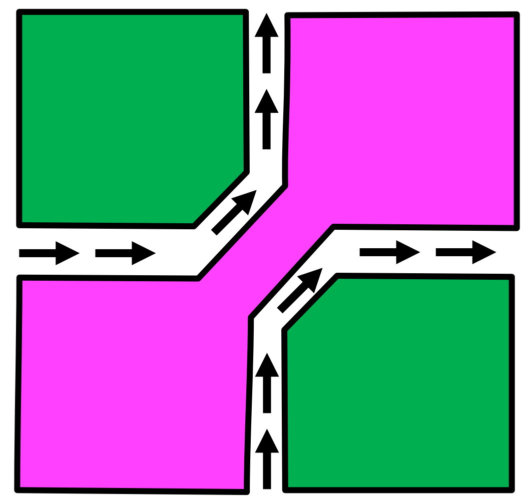
PRL (2014)



What happens in this case?

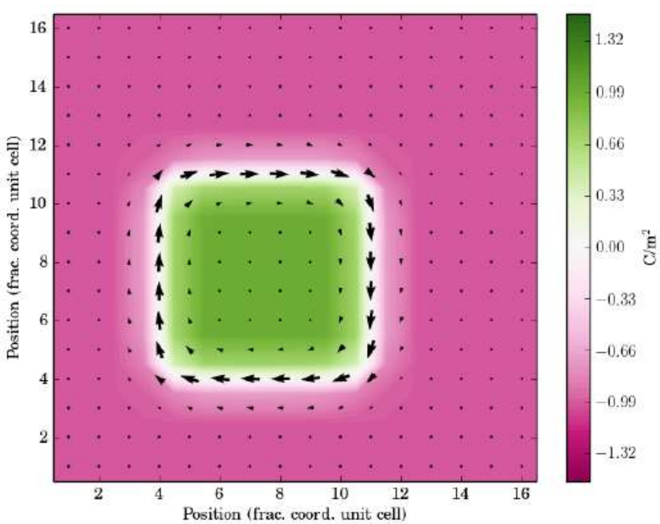


relaxes to…

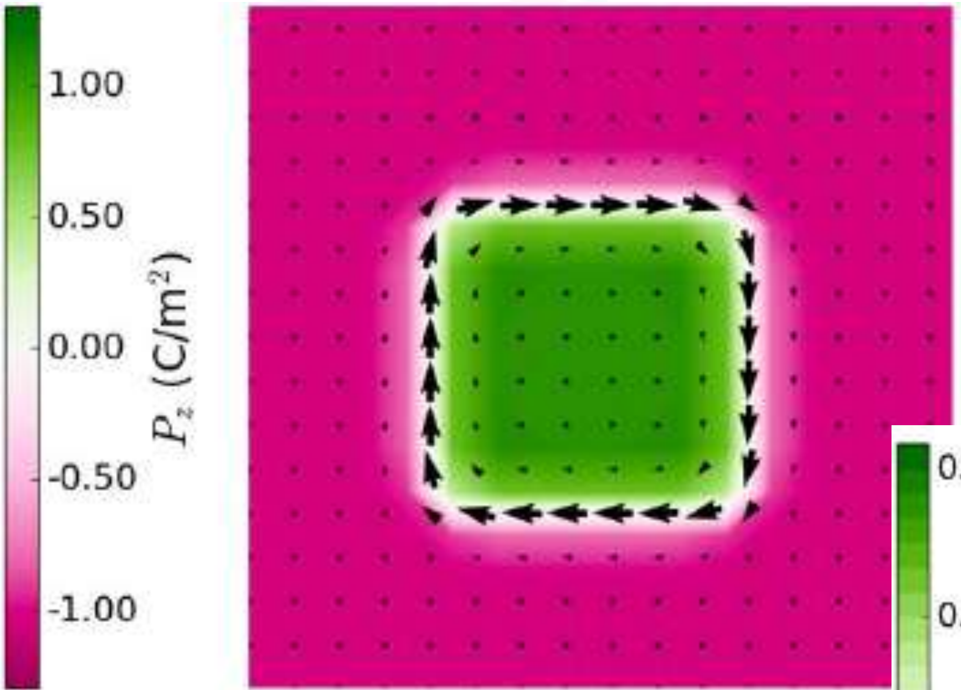


closed DW → skyrmion!

Sci. Adv. (2019)

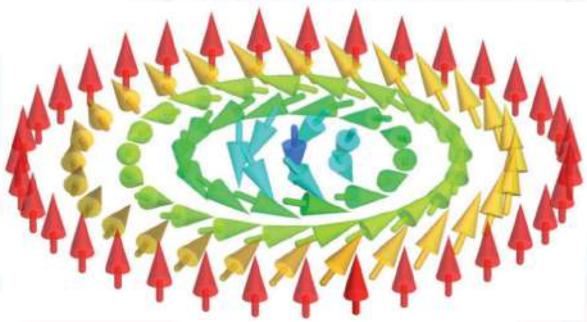


It is a stable Bloch "skyrmion bubble"

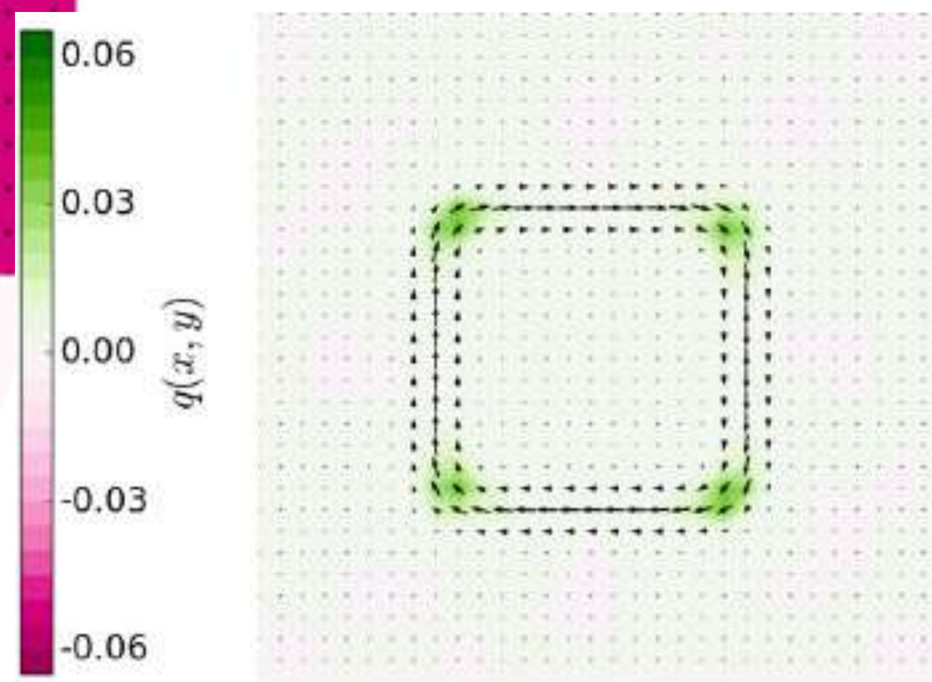


$$Q = \frac{1}{4\pi} \iint d^2\vec{r} \quad \vec{u} \cdot (\partial_x \vec{u} \times \partial_y \vec{u})$$

Pontryagin density *q(x,y)*

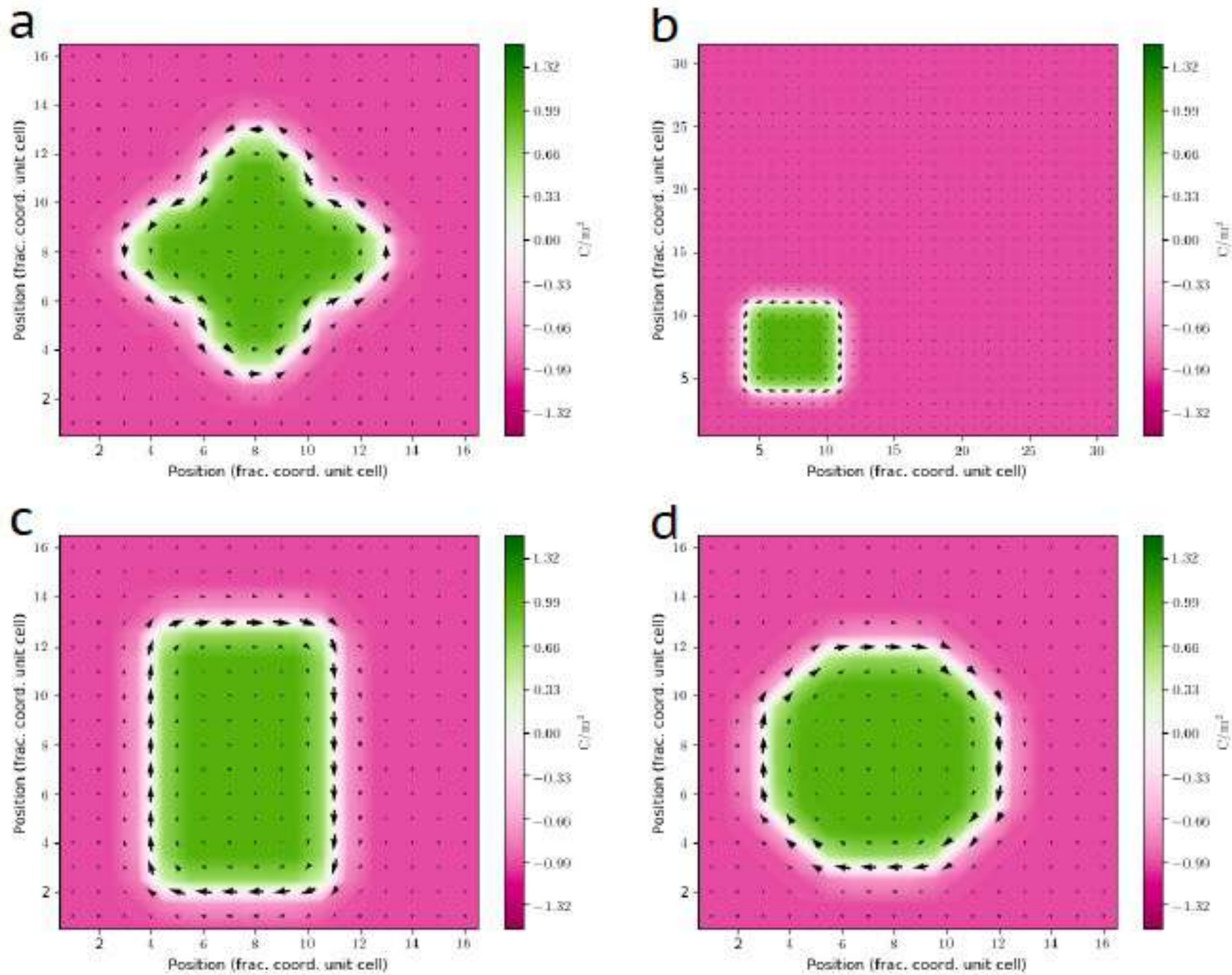


Topological charge Q = 1

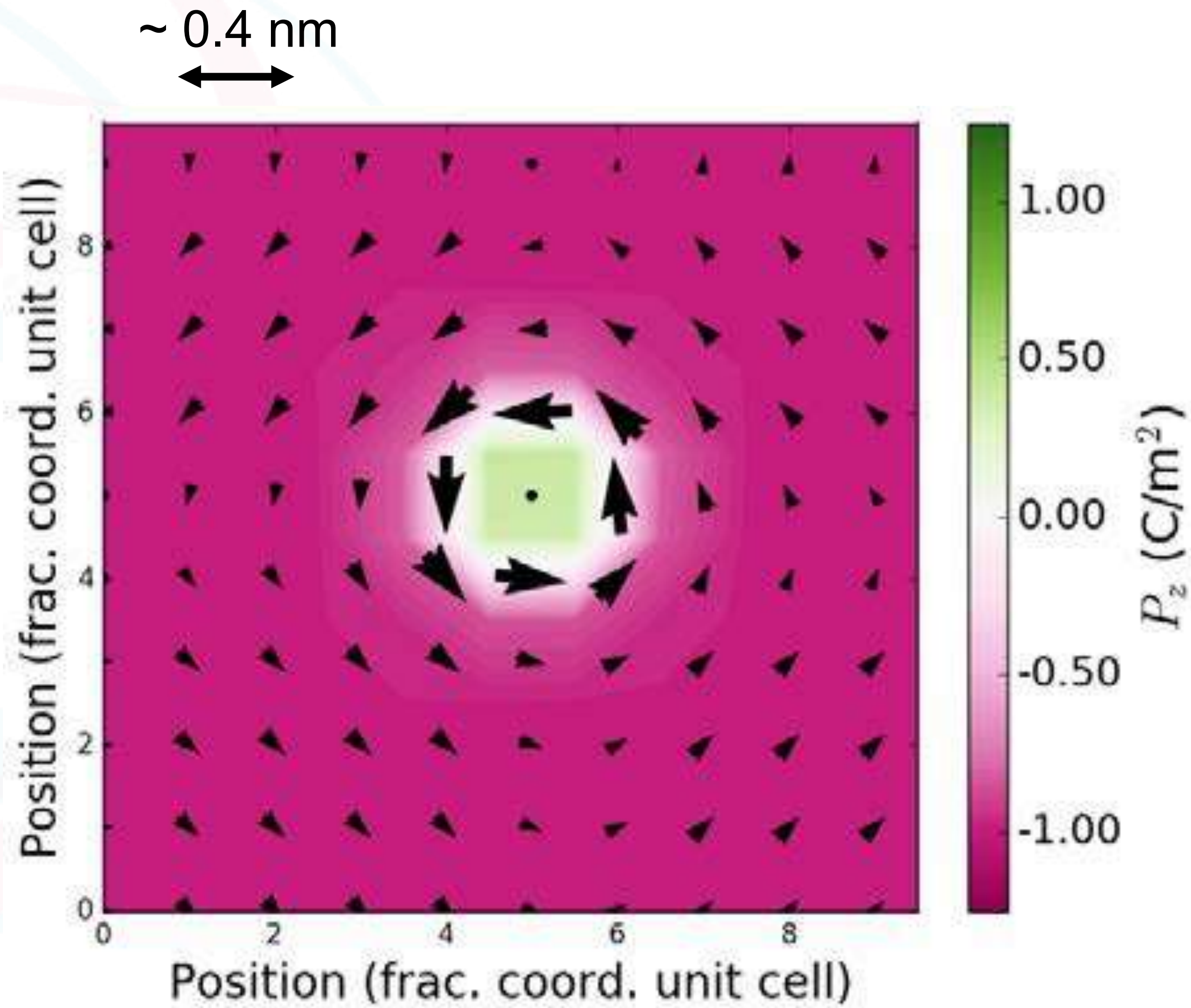


Gonçalves *et al*., Sci. Adv. 5, eaau7023 (2019)

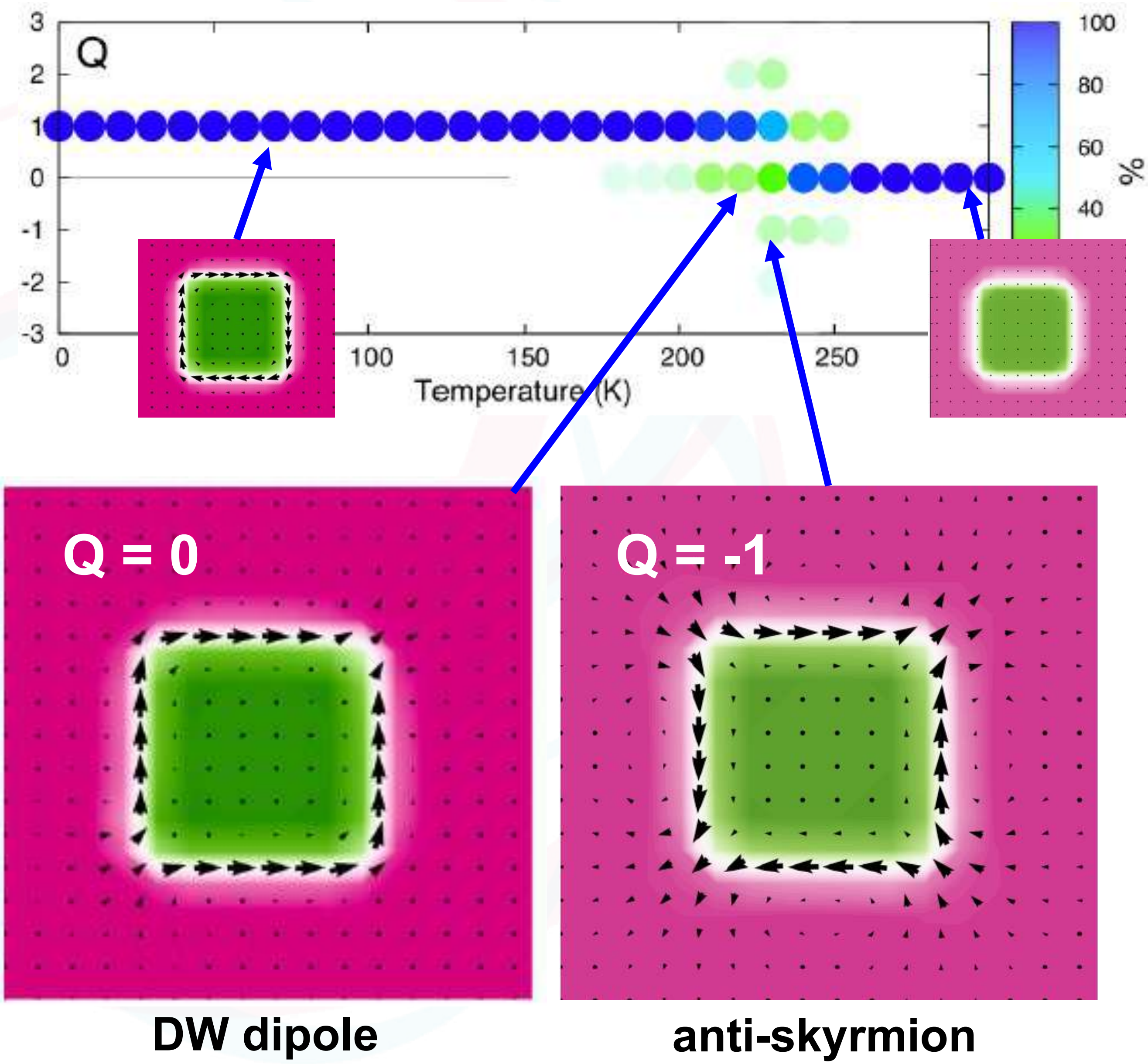
It is a stable Bloch “skyrmion bubble”



And it can be really very small !

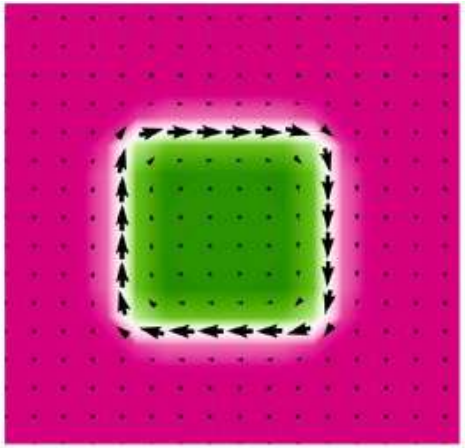


Upon heating: Q = 1 → Q = 0



1. Run Monte Carlo
2. Take snapshots
3. Cut 2D slice
4. Compute Q
5. Construct histogram

Topologically equivalent to…

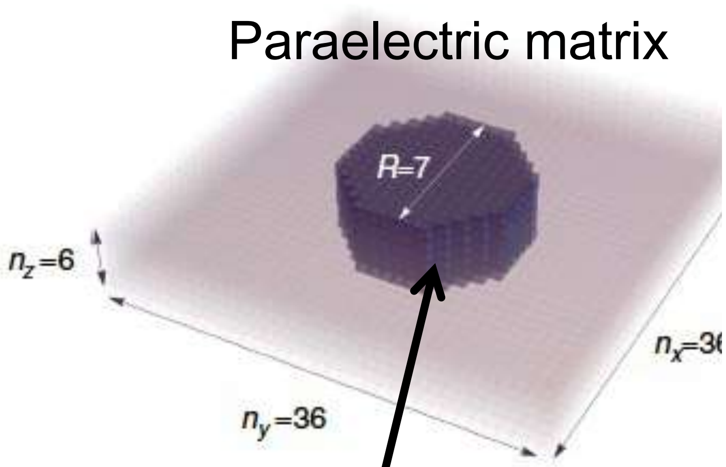


~

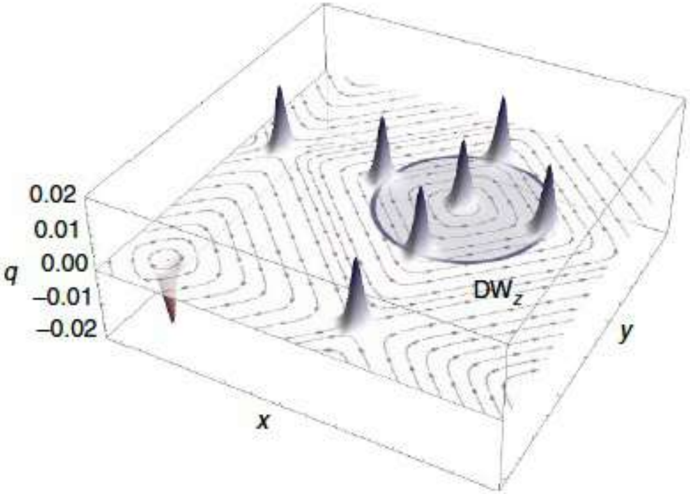
+ out-of-plane polarization

Nahas, Bellaiche *et al.*,
Nat. Comm. 6, 8542 (2016)

Paraelectric matrix



Ferroelectric nano-pilars

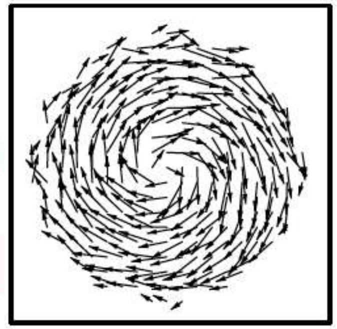


Topological charge

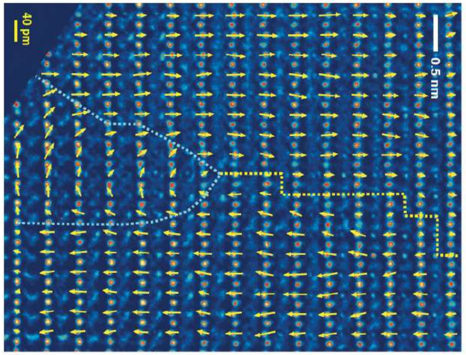
Theory vs Experiment

Non-collinear electric dipoles

2004

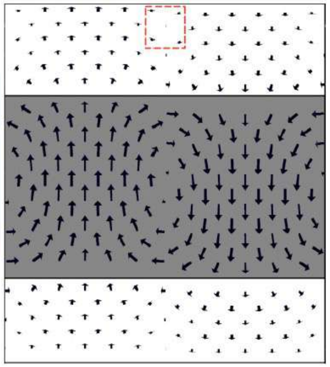


2011

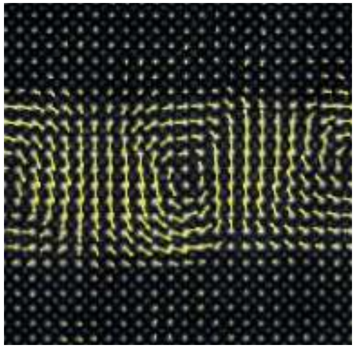


Vortex-like domain walls

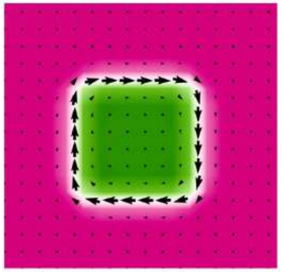
2012



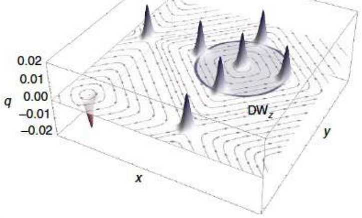
2016



Electric skyrmion bubble



2016 – 2019

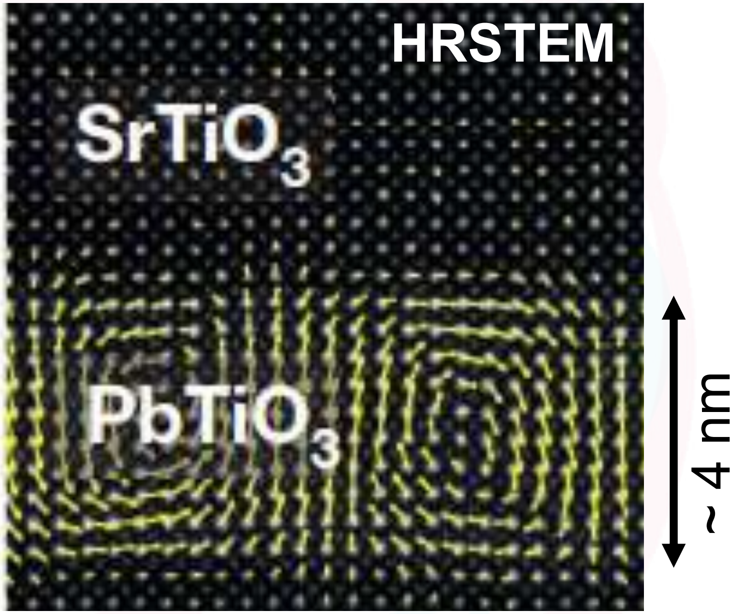


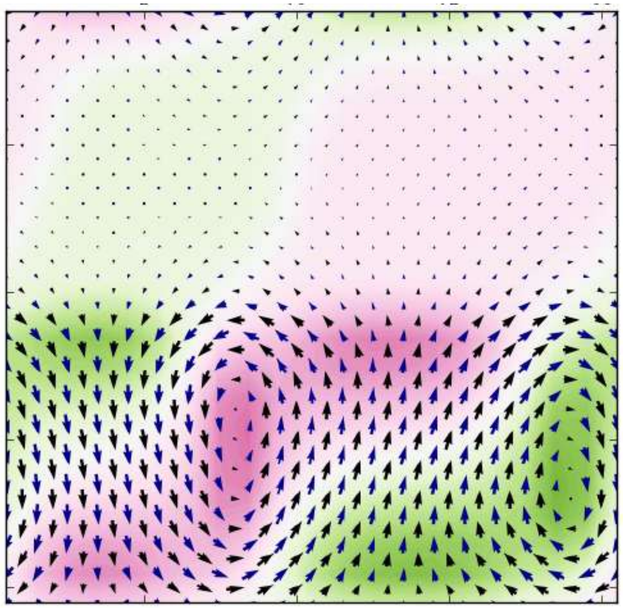
?

Any experimental evidence for DW polarization?

- Key feature (polarization at DWs of $PbTiO_3$) is solid from first principles
- Temperature scale: main issue with our predictions
- Very difficult to measure directly

Yadav *et al.*, Nautre 530, 198 (2016)



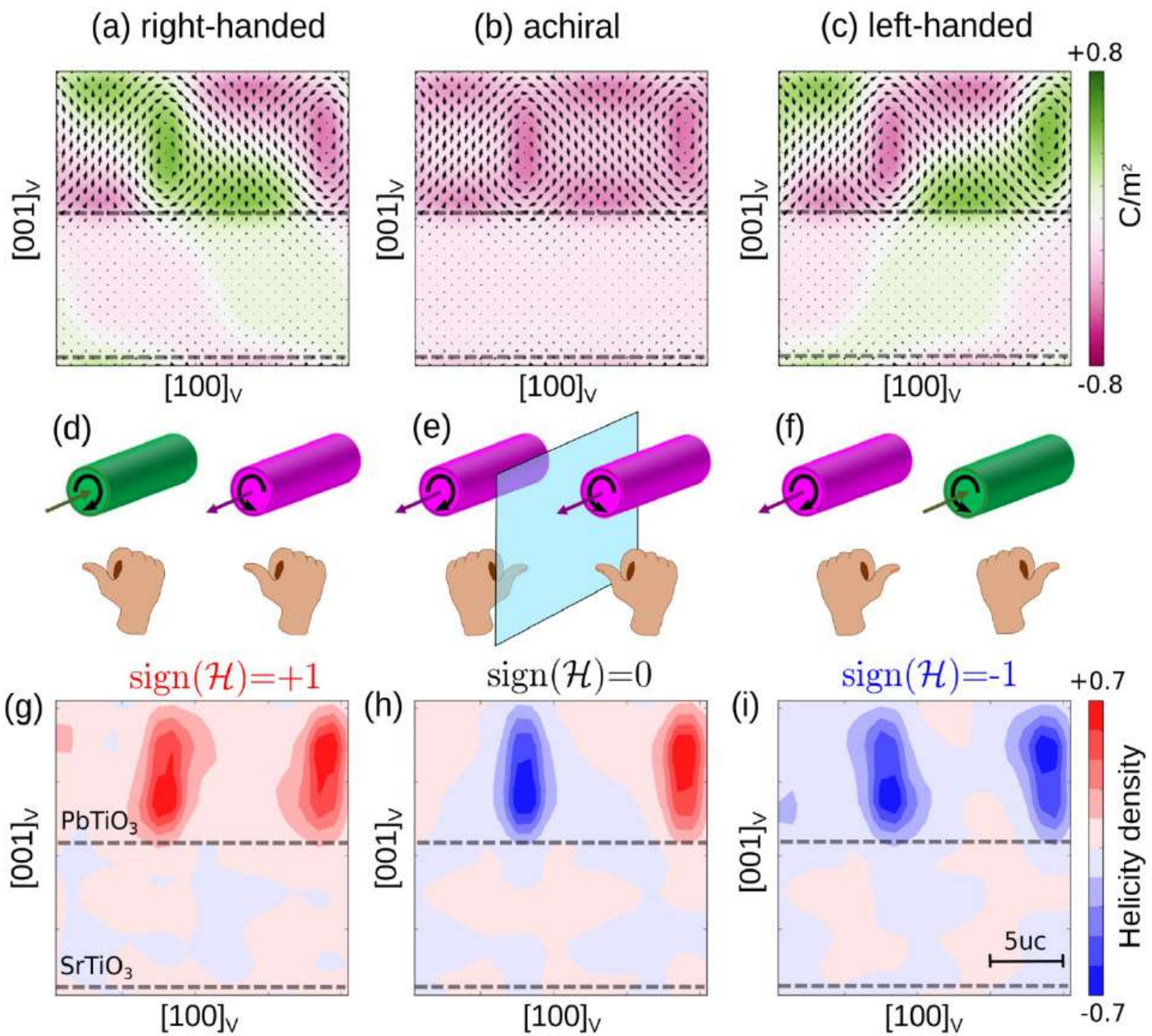


Our model

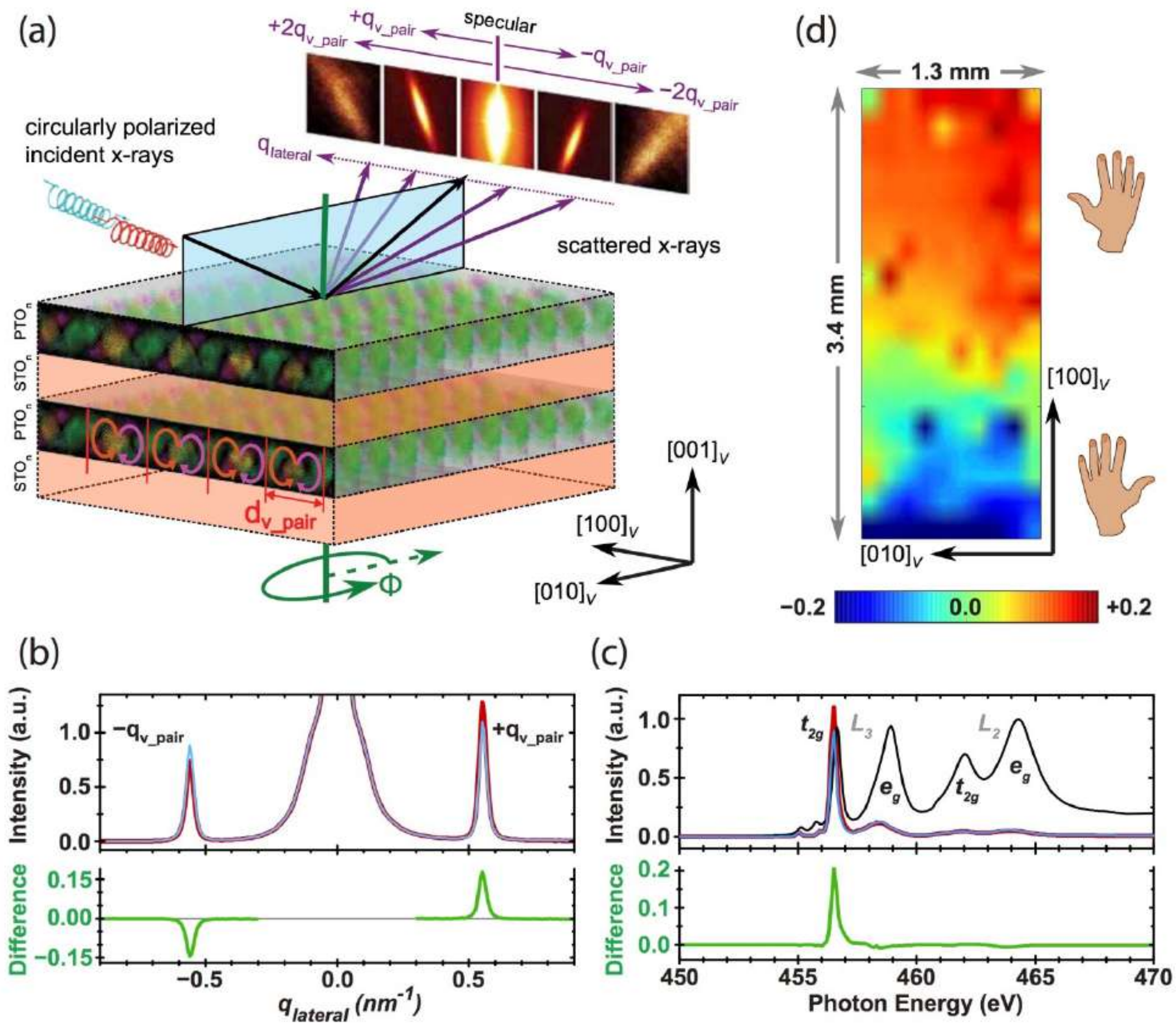
***Emergent chirality in the electric polarization texture of titanate superlattices*, Shafer *et al.*, PNAS 115, 915 (2018)**

- Damodaran *et al.*, Nat. Mats. 16, 1003 (2017)
- Zubko *et al.*, PRL 104, 187601 (2010); Aguado-Puente *et al.*, PRB 85, 184105 (2012)

Chirality in $PbTiO_3/SrTiO_3$ superlattices



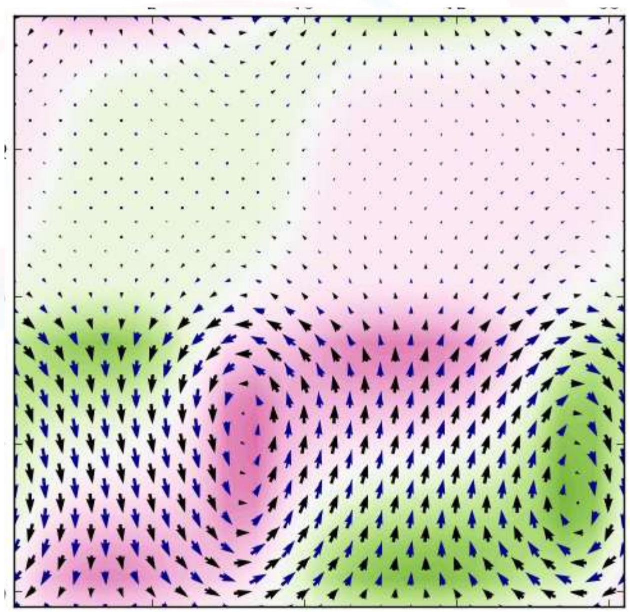
Chirality in $PbTiO_3/SrTiO_3$ superlattices

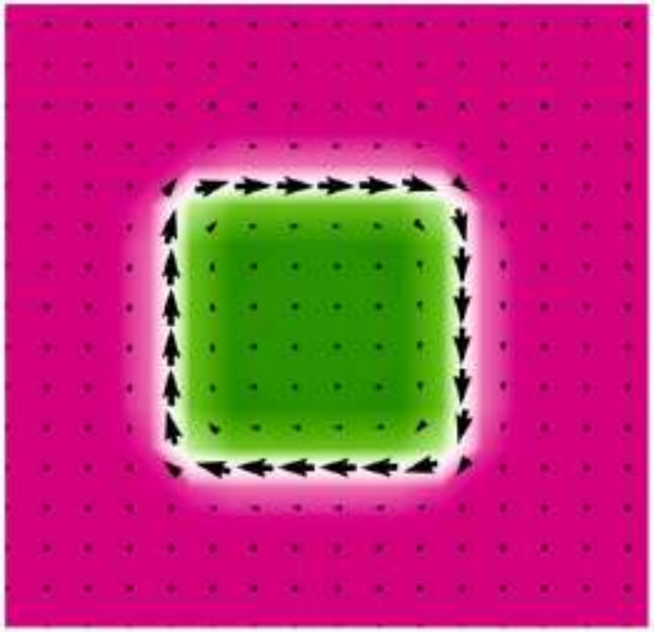


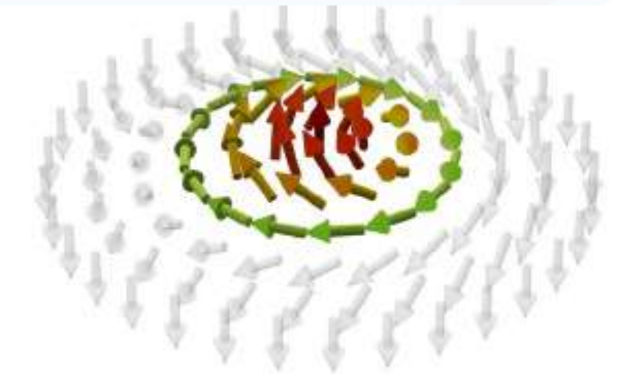
Experimental evidence of from resonant soft x-ray diffraction

We are almost there, but…

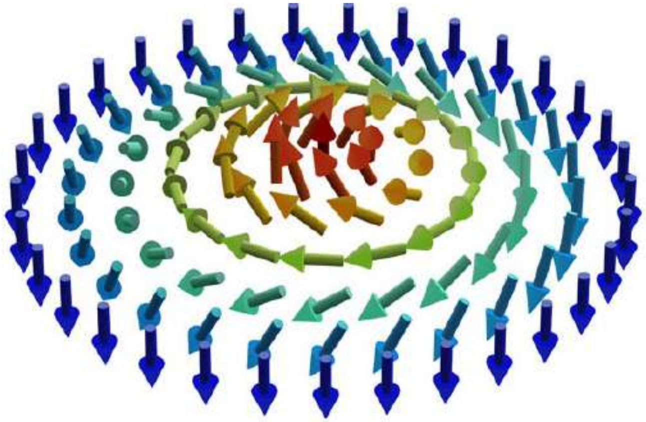
Is there a skyrmion here?







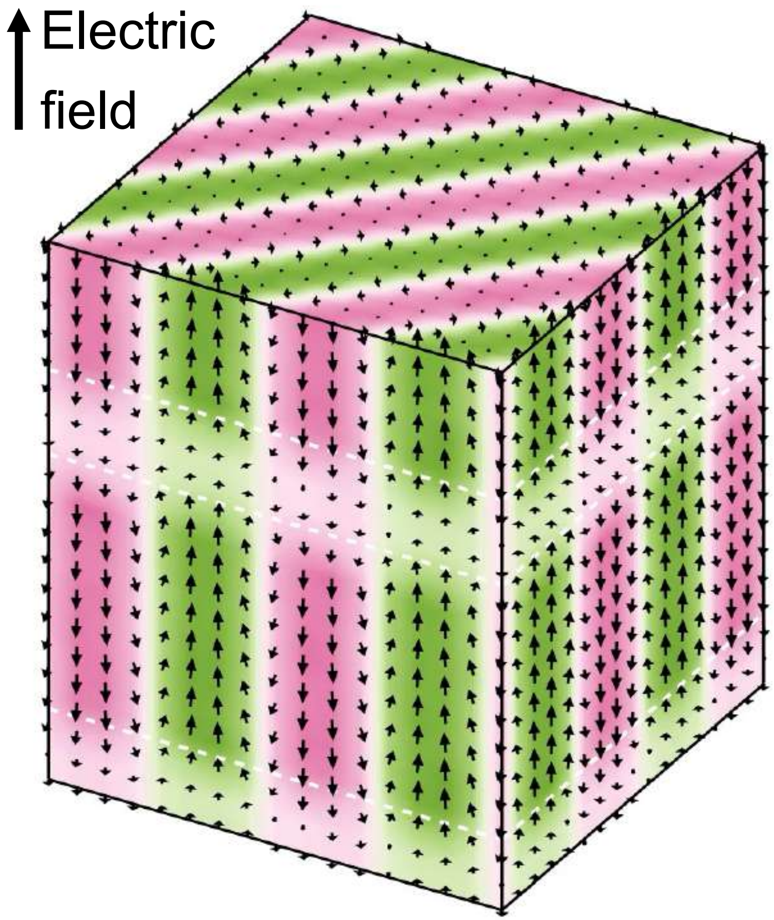
In fact, the "polarized vortexes" are "**merons**" with Q=1/2



[Meron figure taken from Shao *et al*., Nat. Comm. 14, 1355 (2023)]

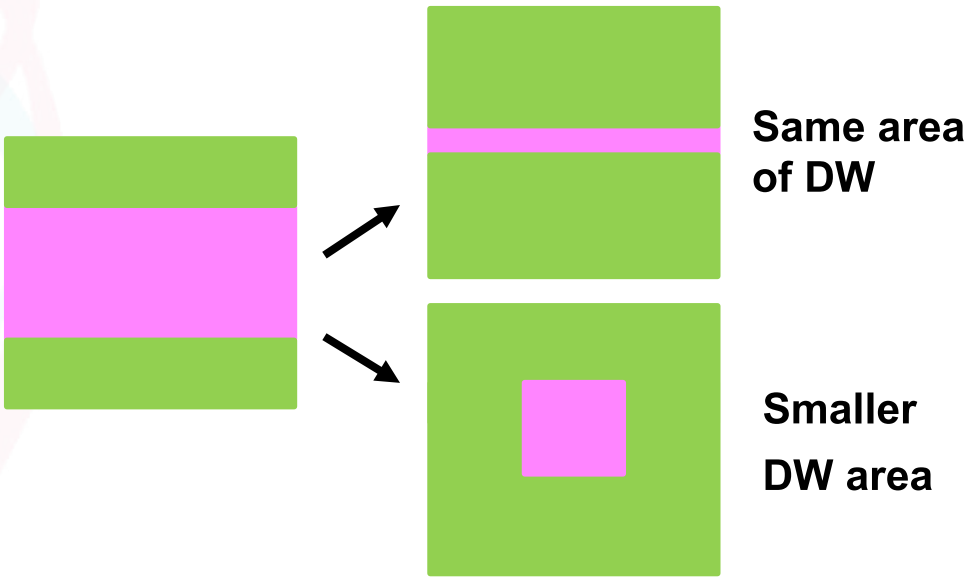
Questions?

The last step: breaking the stripes



Stripes → Bubbles

- Why bubbles under fields?
  → Minimize domain-wall energy

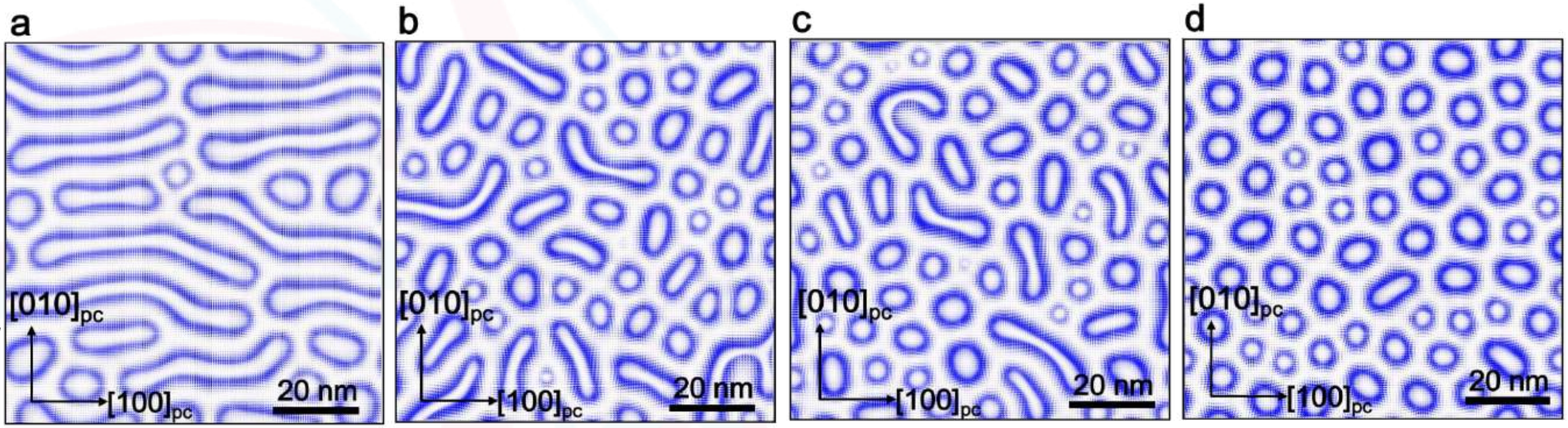


(Plus: bubbles favored by entropy)

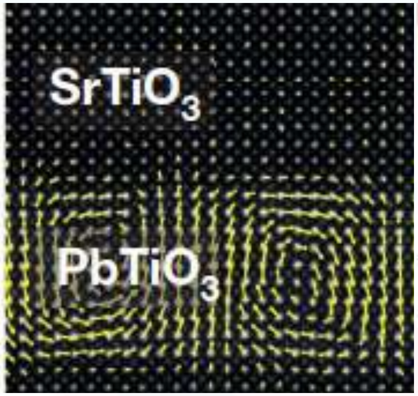
Aramberri & Íñiguez (2023), https://arxiv.org/abs/2308.01716

A jungle of states to explore

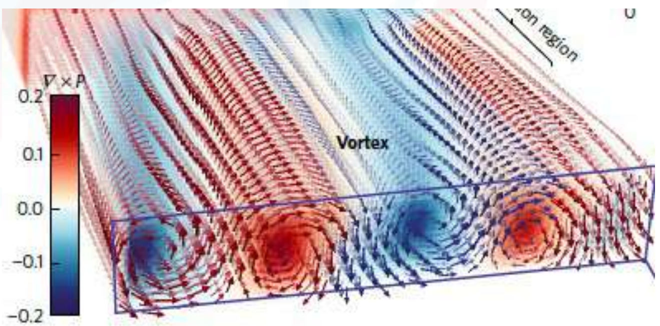
stripes → **bubbles**



Phase field simulations (Hong *et al*.) for various STO/PTO/STO tri-layers and PTO/STO superlattices; many closely competing phases; consistent with Triscone *et al*., Bellaiche *et al*., Zubko *et al*.; Valanoor *et al*.



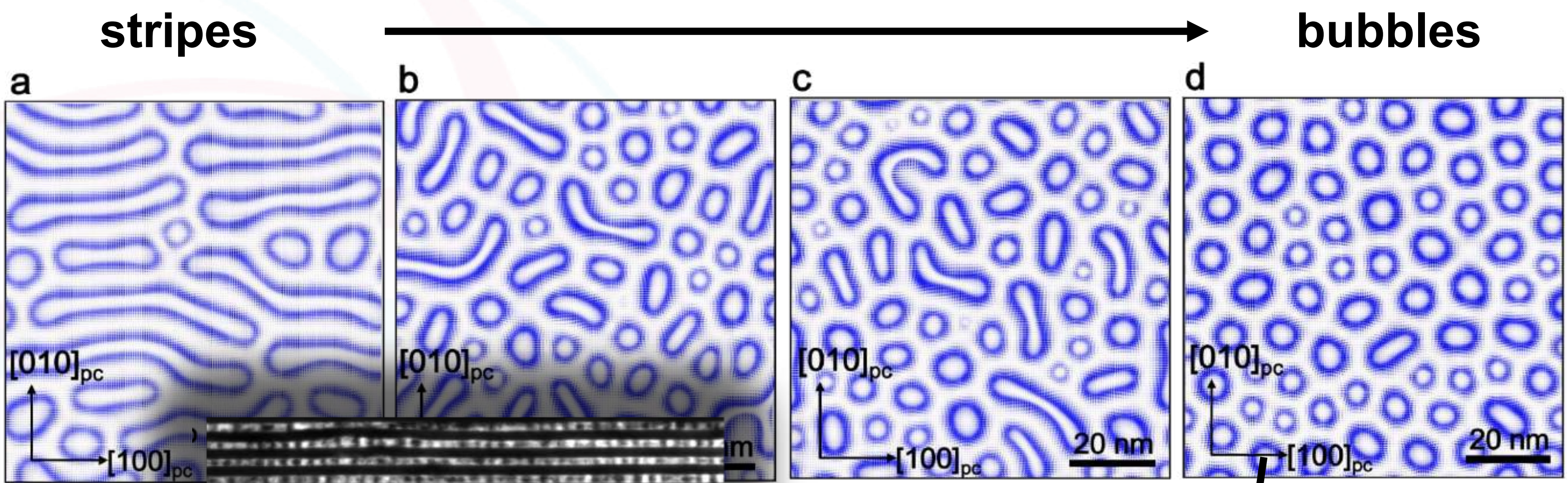
PTO/STO SL on $DyScO_3$



Yadav *et al*., Nautre <u>530</u>, 198 (2016)

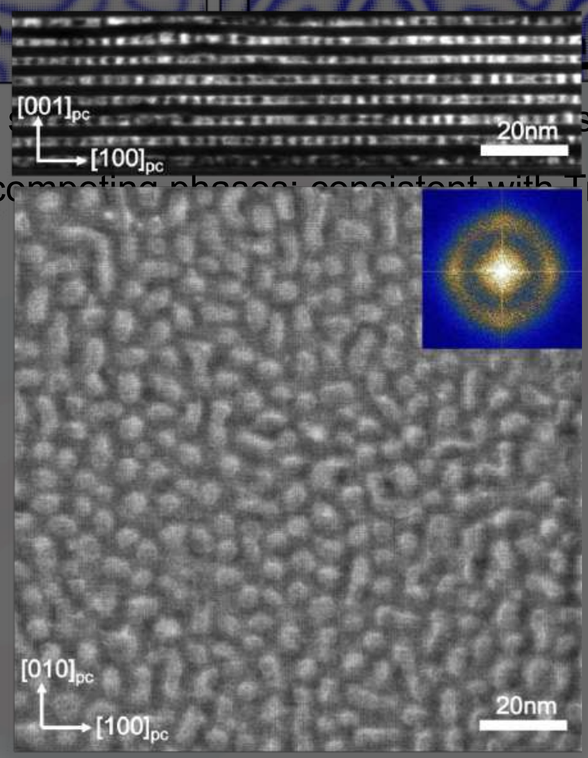
Damodaran *et al*., Nat. Mats. <u>16</u>, 1003 (2017)

A jungle of states to explore



Phase field s… s STO/PTO/STO tri-layers and PTO/STO superlattices;

many closely c… Triscone *et al*., Bellaiche *et al*., Zubko *et al*.; Valanoor *et al*.

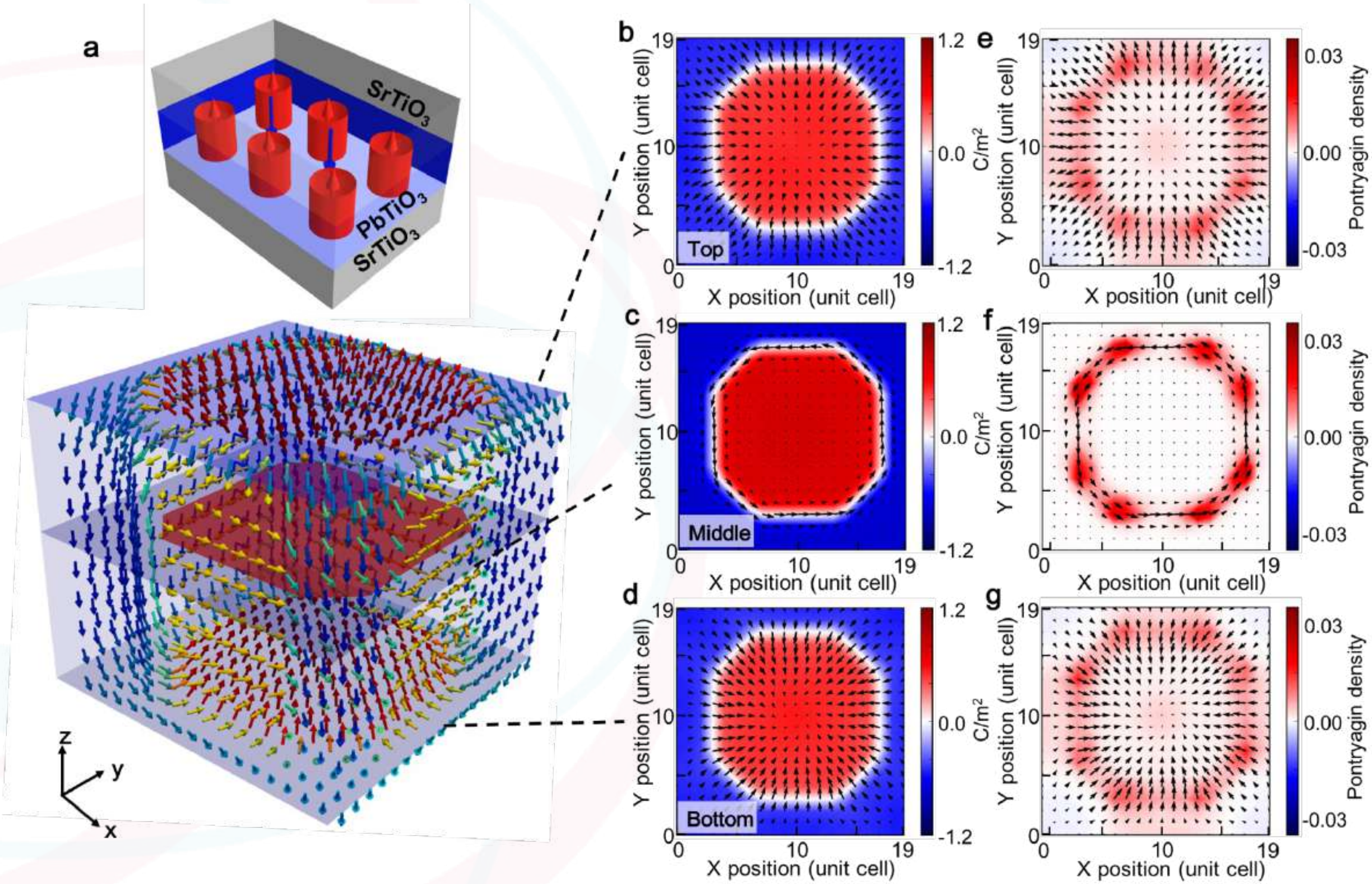


PTO/STO SL on $SrTiO_3$

Das *et al*., Nature 568, 368 (2019)

Second-principles simualtions

Das *et al*., Nature (2019)



Néel

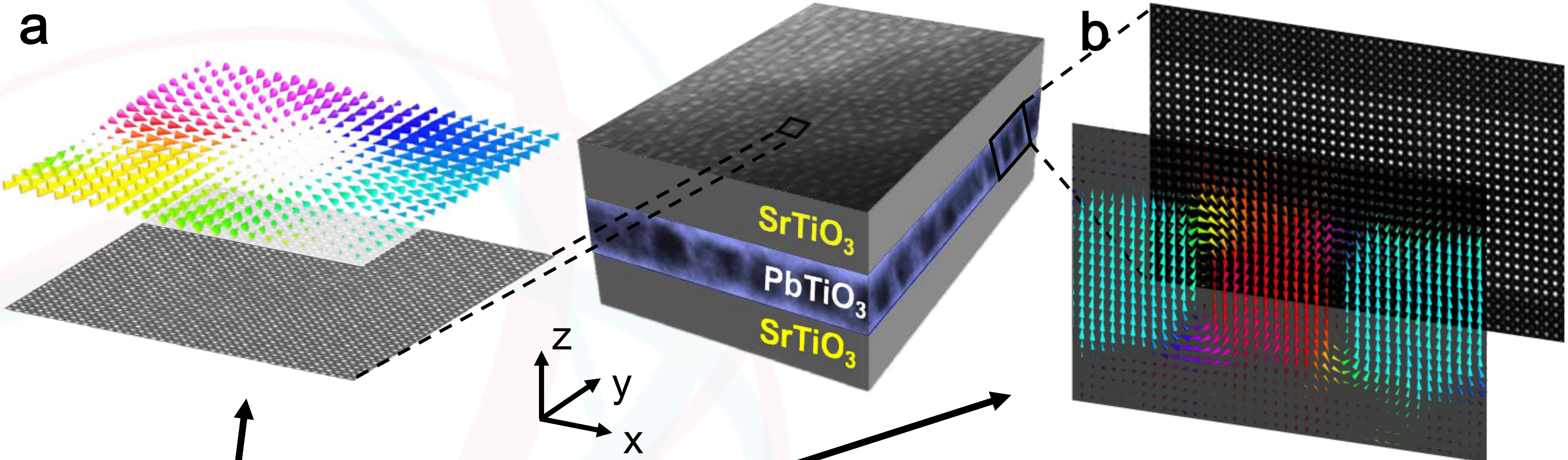
Bloch

Néel

- Topological number is Q=1 all along the column domain
- Character varies (Néel → Bloch → Néel) as we move along

STEM studies of PTO/STO on STO

Das *et al*., Nature (2019)

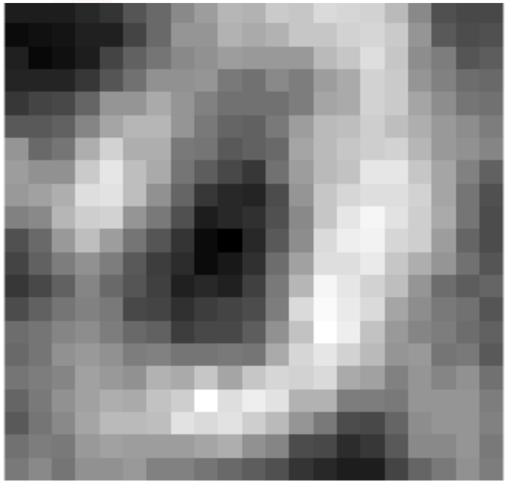


HAADF-STEM (@ Berkeley)

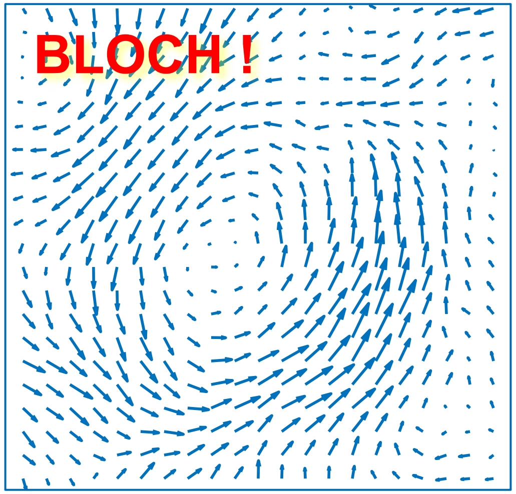
beam convergence angle: 17 mrad

4D-STEM (@ Cornell, Nguyen, Muller's group)

beam convergence angle: 1.7 mrad

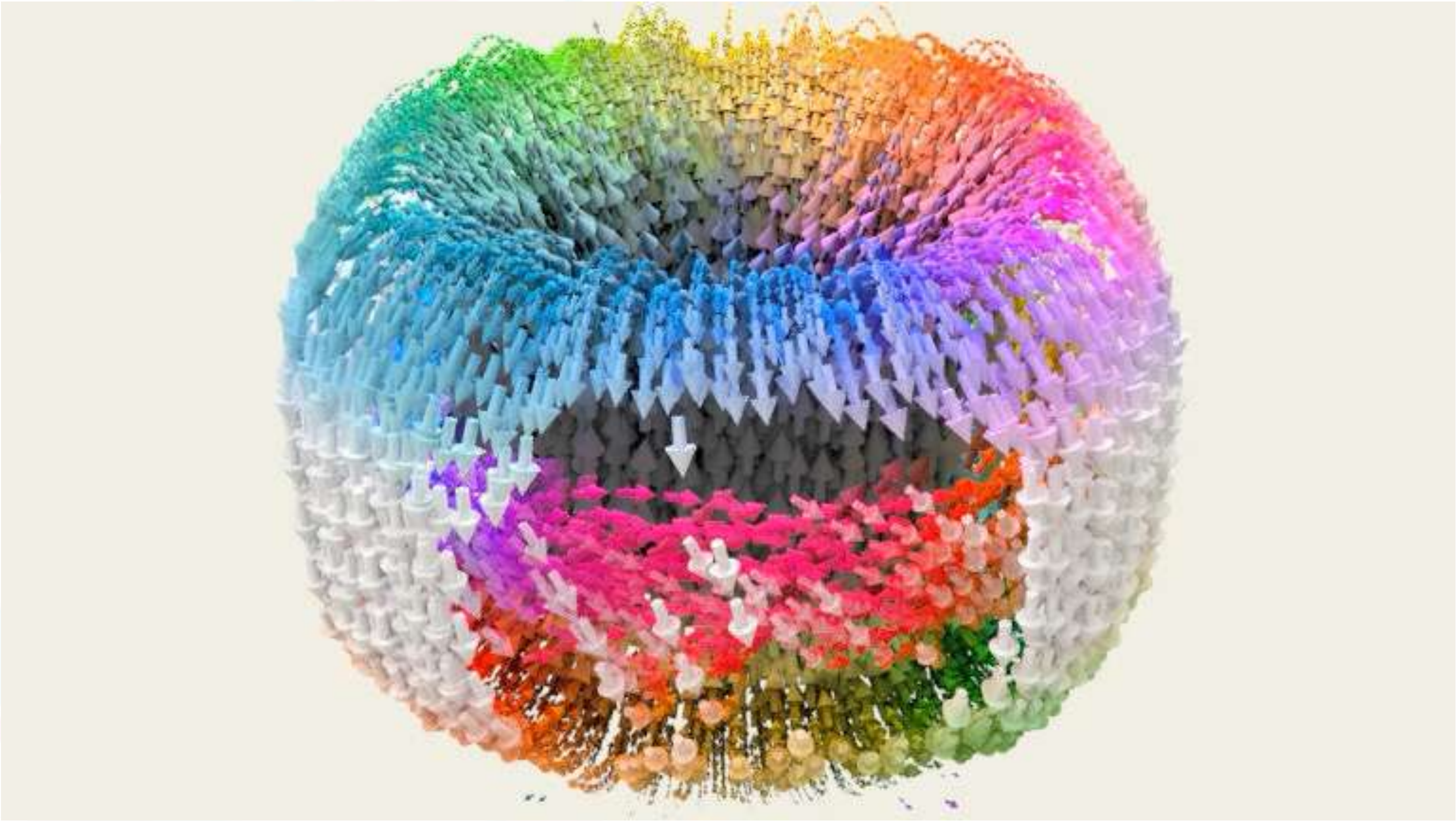


ADF image



in-plane dipoles

A little movie from the simulation data

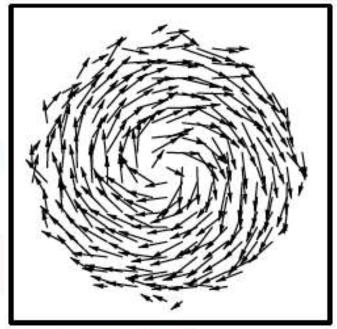


Das *et al.*, Nature (2019)

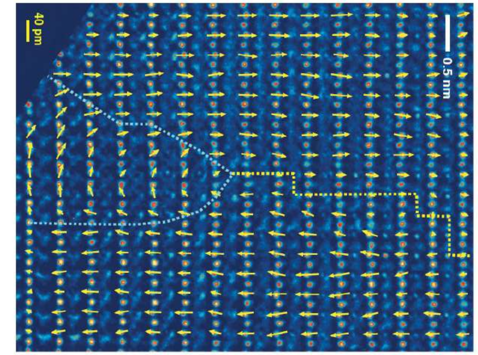
Theory vs Experiment

Non-collinear electric dipoles

2004

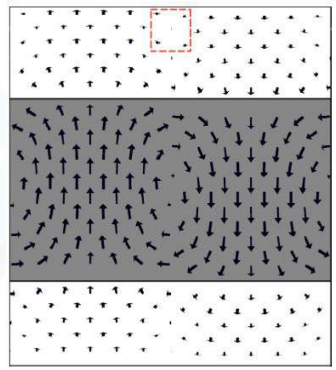


2011

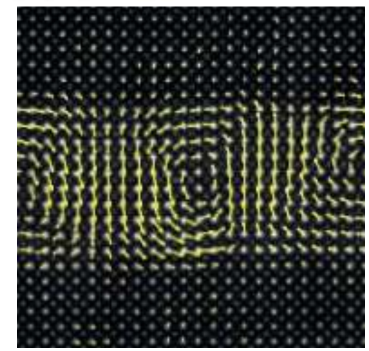


Vortex-like domain walls

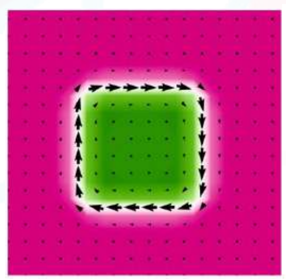
2012



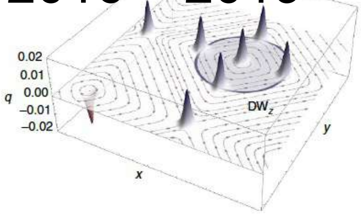
2016

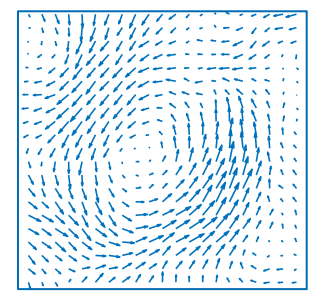


Electric skyrmion bubble



2016 – 2019

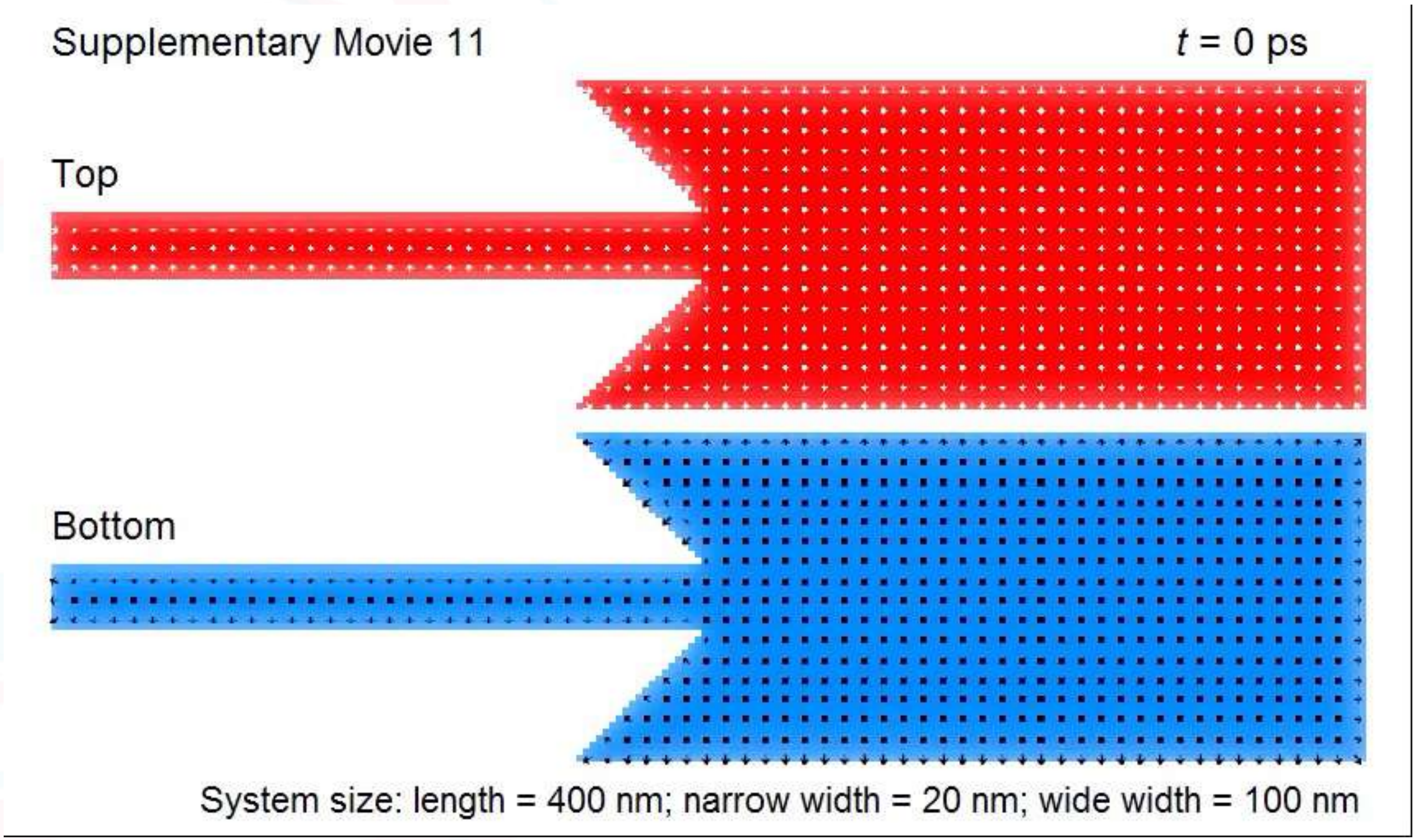




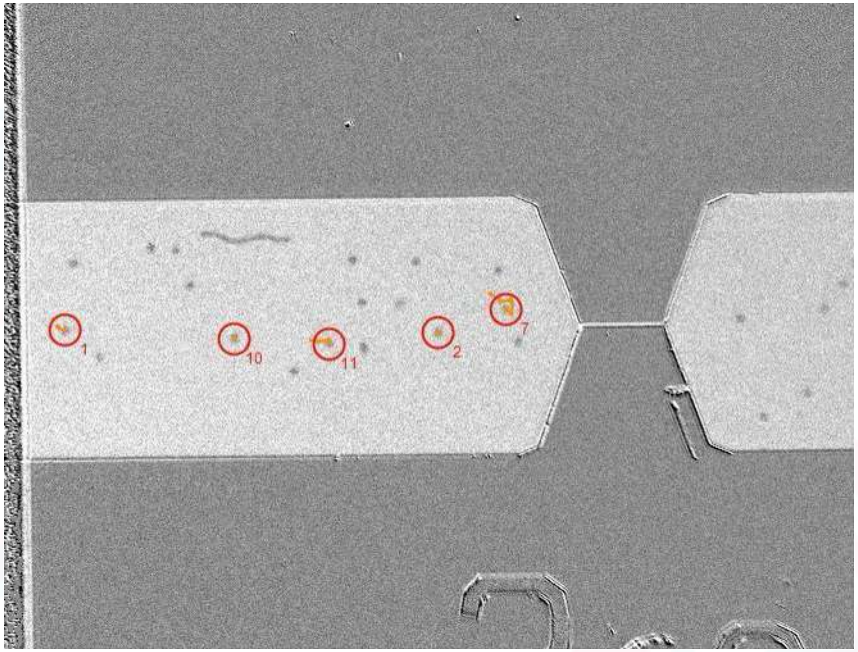
2019

Questions?

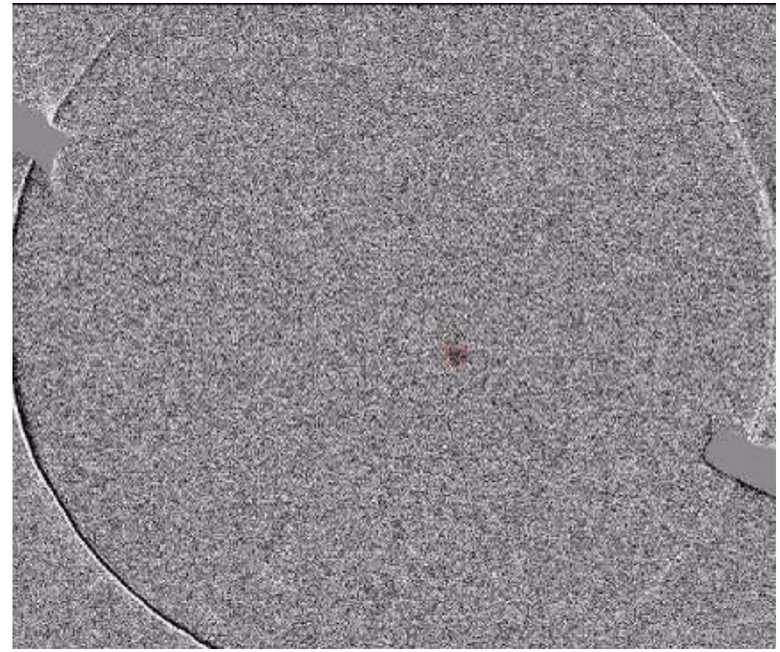
What strikes you about these images?



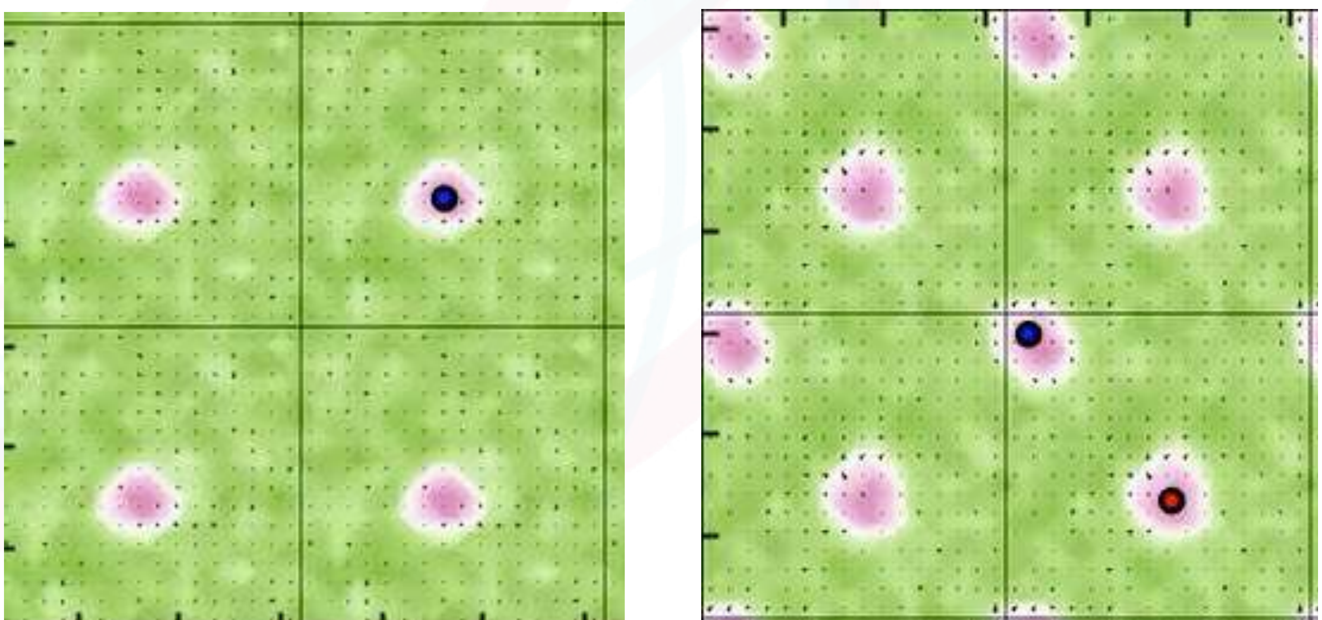
Zhang *et al*., Nature Communications **7**, 10293 (2016)



Zázvorka *et al*., Nat. Nano 14, 658 (2019)



Zhao *et al*., PRL 125, 027206 (2020)

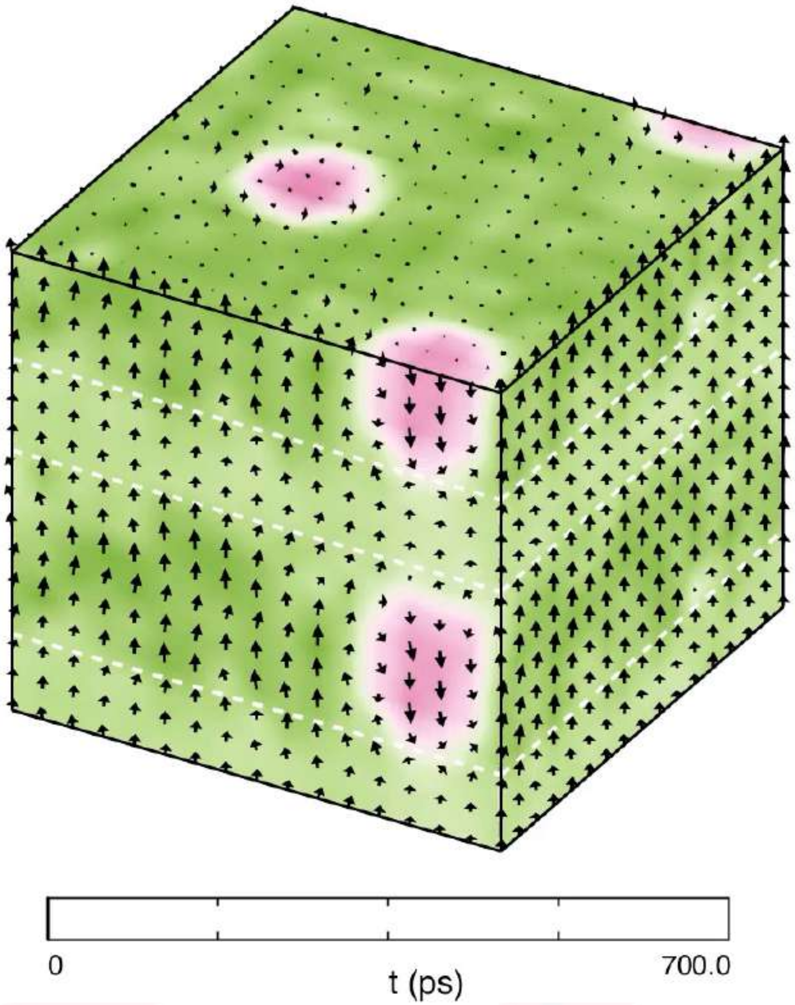


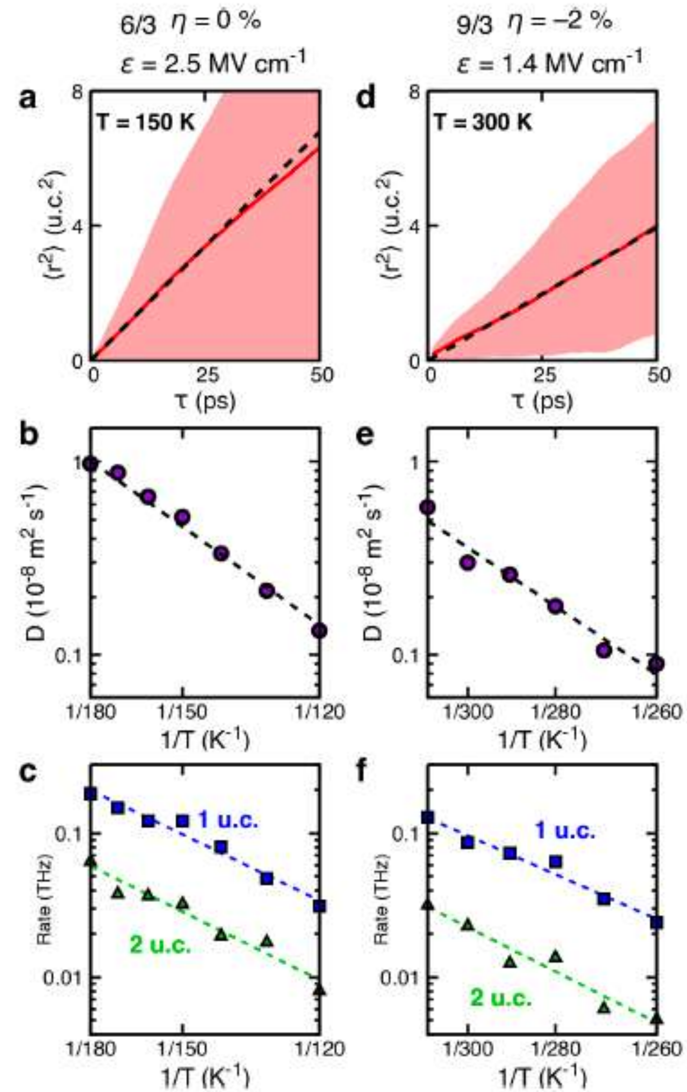
Aramberri & Íñiguez, arXiv (2023)

6/3 SL @ 150 K

~ 2 MV/cm

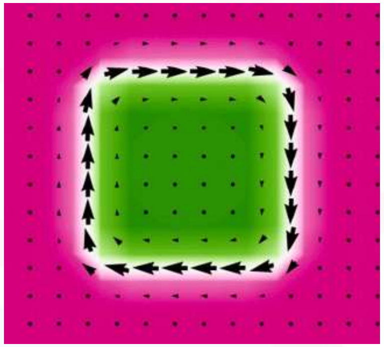
Brownian dynamics of electric bubbles

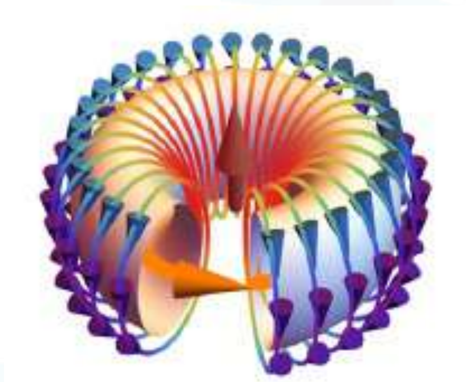


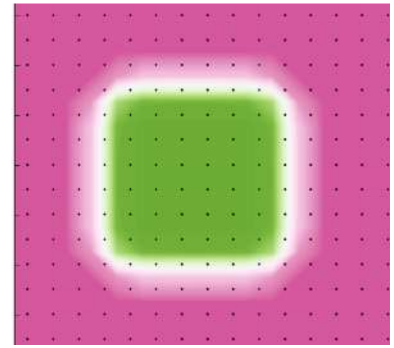


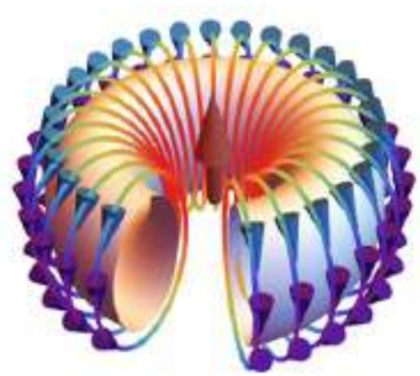
Aramberri & Íñiguez (2023), https://arxiv.org/abs/2308.01716

Are these particles “topological”?









Low temperature

Static

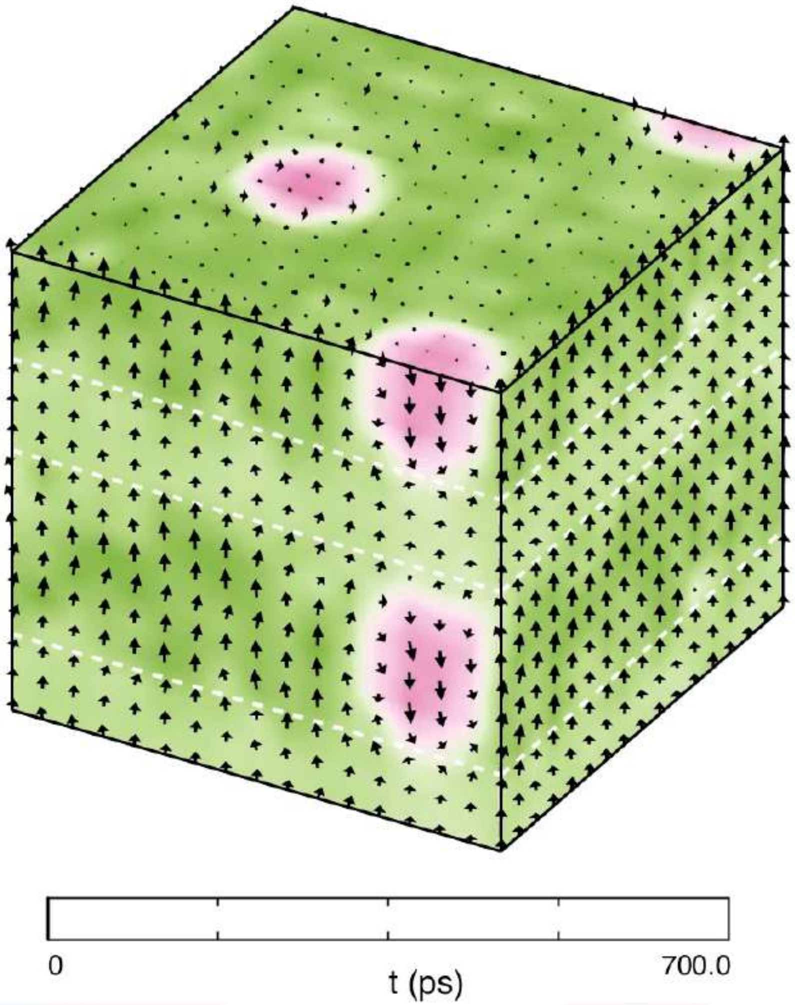
Bloch & Nèel features

High temperature

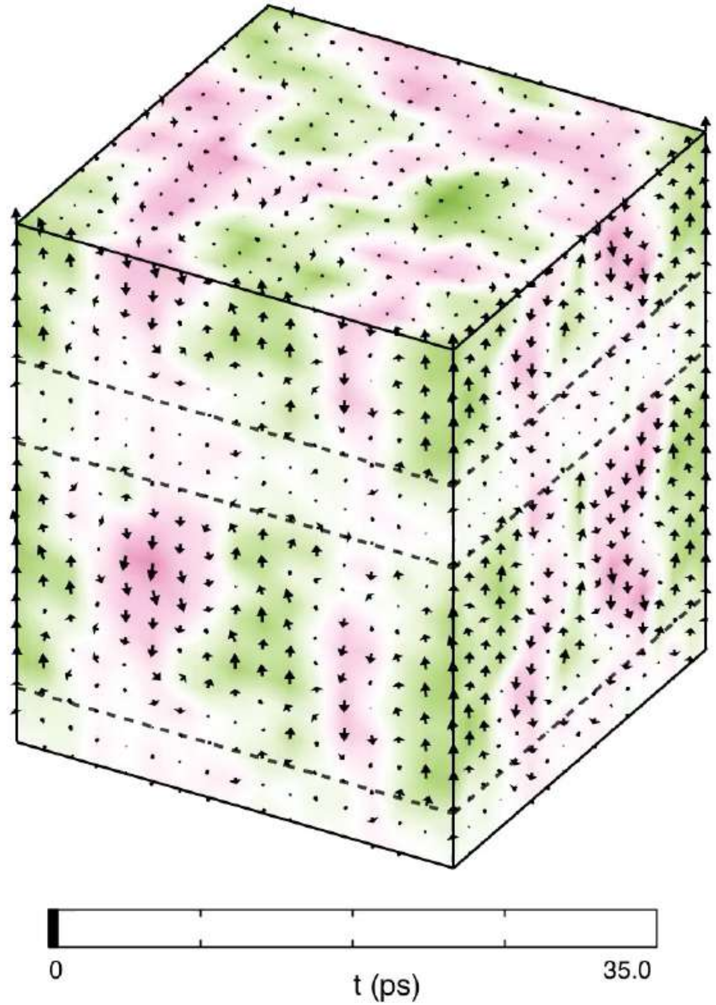
Mobile

Only Nèel features

Brownian dynamics only for bubbles?



Aramberri & Íñiguez (2023)



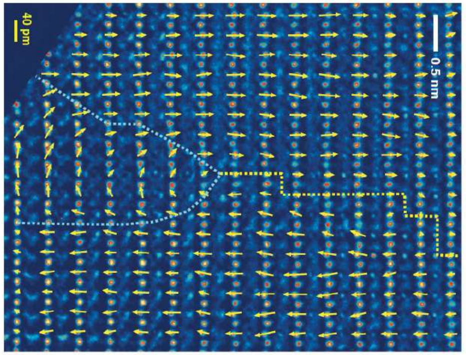
Zubko *et al*. (2016)

Theory vs Experiment

Non-collinear electric dipoles

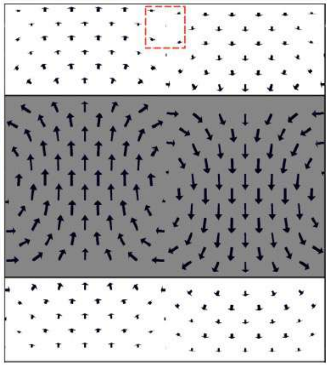
2004

2011

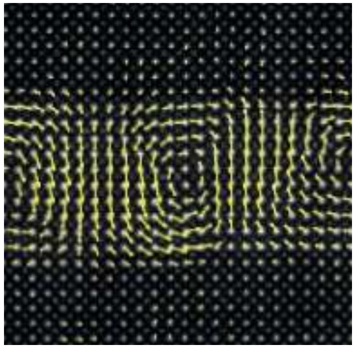


Vortex-like domain walls

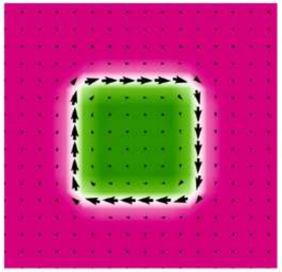
2012



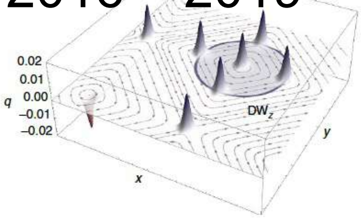
2016

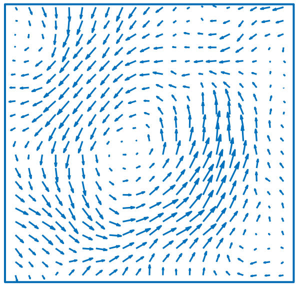


Electric skyrmion bubble



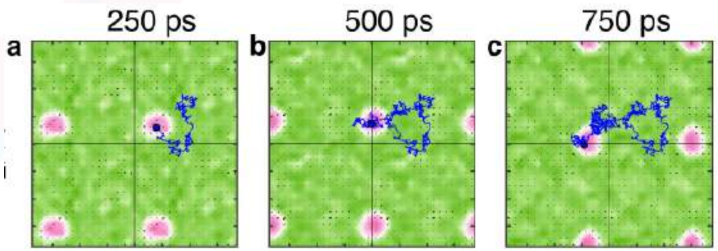
2016 – 2019





2019

Electric bubble quasiparticle



2023

….

So, how big a deal is this?

- **If you are interested in chirality, toroidicity, etc.**
  - → Require arrangements typical of skyrmions, vortices (topological)
  - → Although similar non-topological arrangements (merons) can also present such properties. Shafer (2018) Das (2019)
- **If you are interested in (other) functional properties…**
  - → **Negative capacitance**: both topo and non-topo Zubko (2016) Das (2021)
  - → **Tunable ~ THz**: non-topo ✔ ; topo ⏱ Li (2021)
- **If you are interested in the particle-like behavior**
  - → You definitely want skyrmion-like (i.e., bubble-like) objects…
  - → … which may well be topological Aramberri (2023)

Questions?

Word on the street !

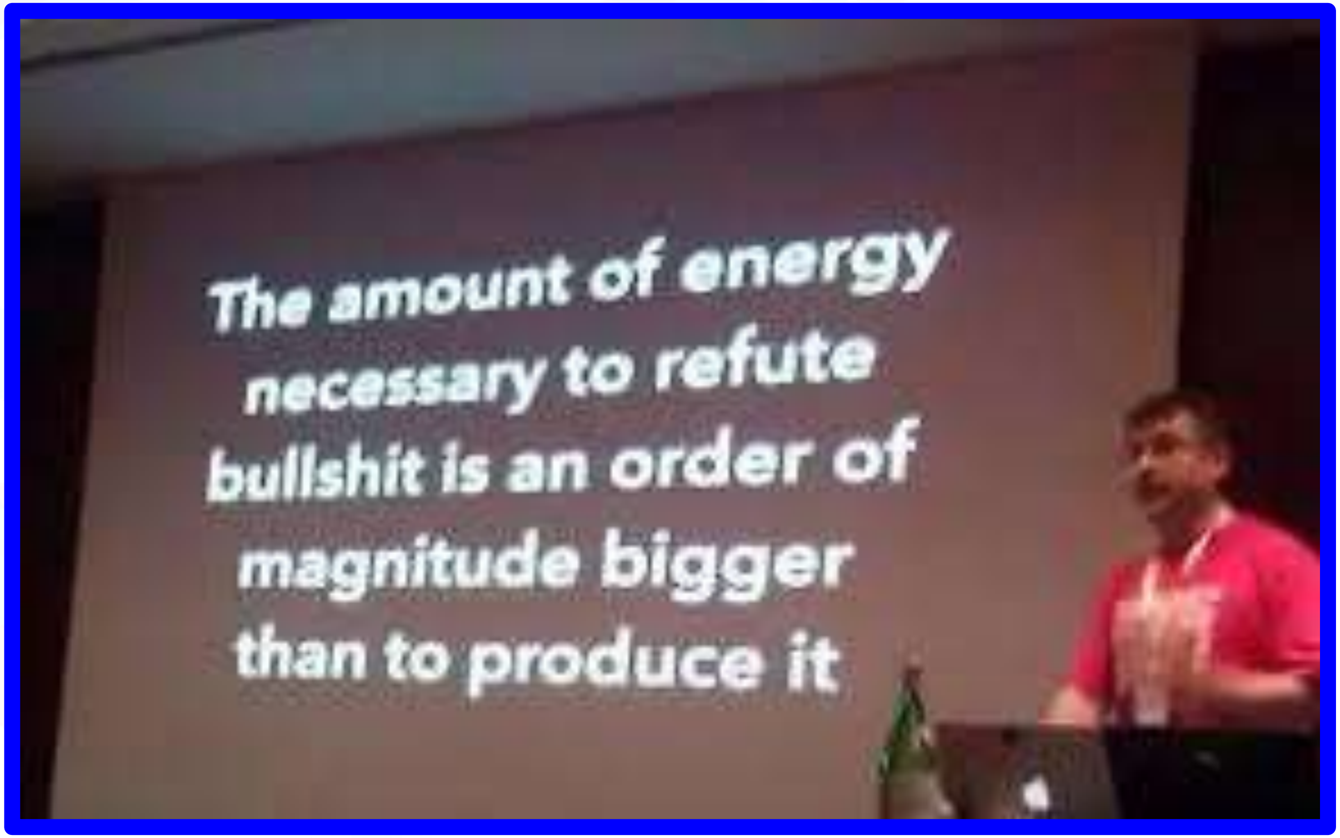
- "All phase transitions involve a change in topology"
- "Skyrmions are very stable because they are topologically protected"
- "Non-topological states cannot be stable"
- "The occurrence of skyrmions requires interactions of the Dzyaloshinsky-Moriya type"
- "All topological states are chiral"
- "All chiral states are topological"
- "Skyrmions and anti-skyrmions cannot coexist (for long times) because they annihilate each other"
- "We can expect many functional properties to be exclusive of electric skyrmions"
- …

Bastiat (1845), Brandolini (2013)

"[…] nos adversaires dans la discussion ont sur nous un avantage signalé. Ils peuvent en quelques mots exposer une vérité incomplète ; et, pour montrer qu'elle est incomplète, il nous faut de longues et arides dissertations."

Frédéric Bastiat, *Sophismes Économiques* (1845)

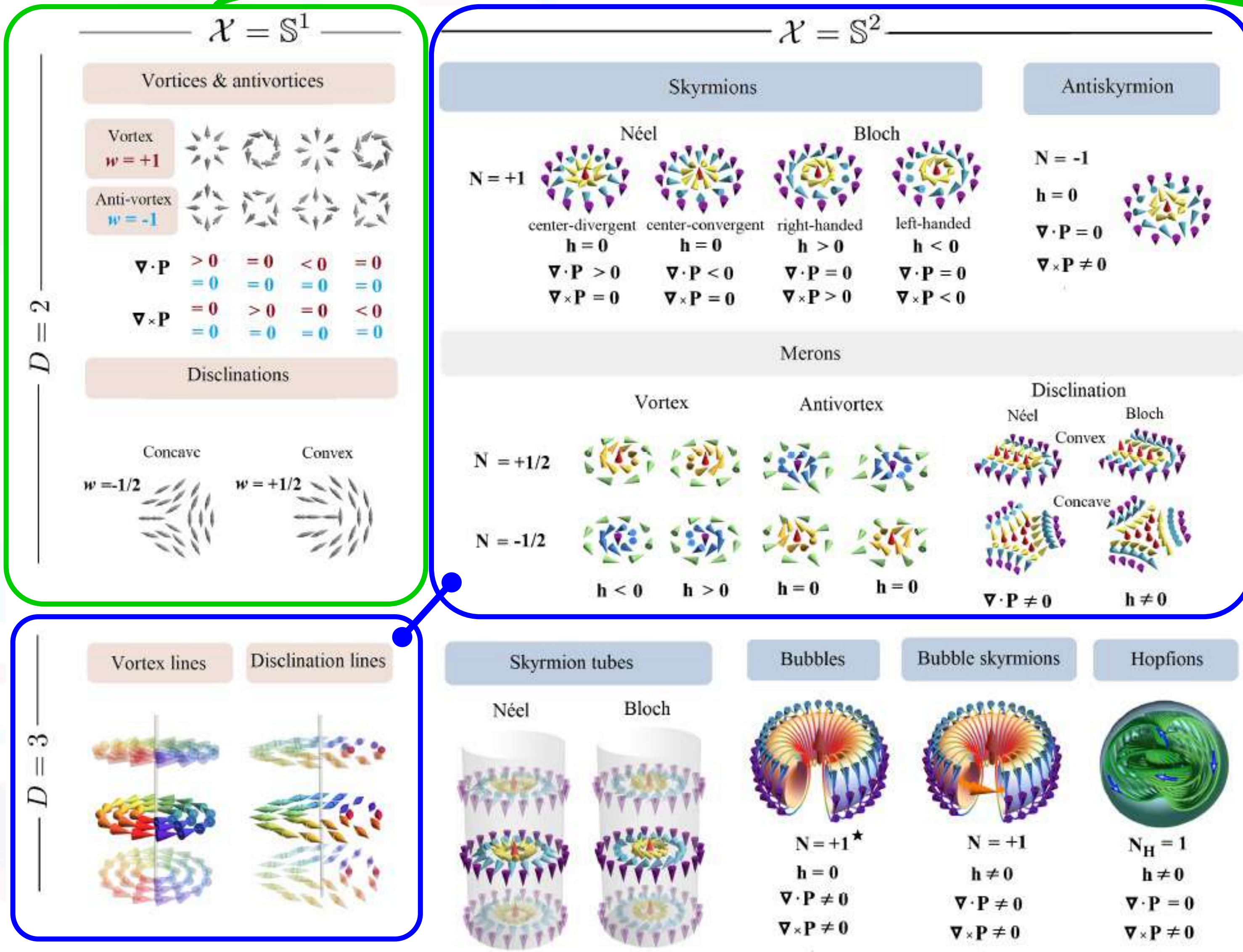
Reported independently in 2013 as the **"bullshit asymmetry principle"**



https://en.wikipedia.org/wiki/Brandolini%27s_law

How to chose your topological invariant?

👉 Dimension of the object (D), of the order parameter space (χ)



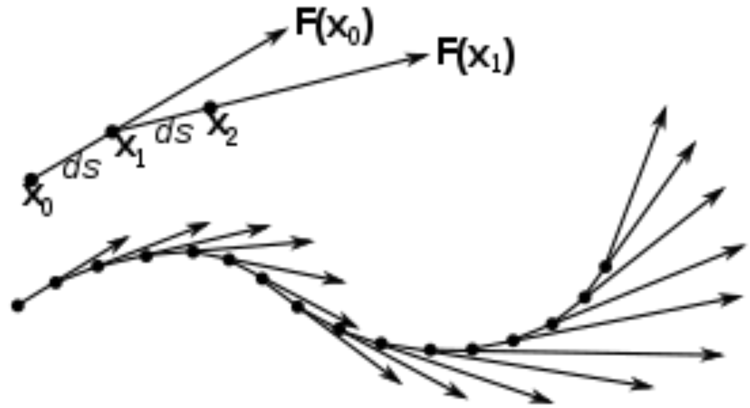
Winding number

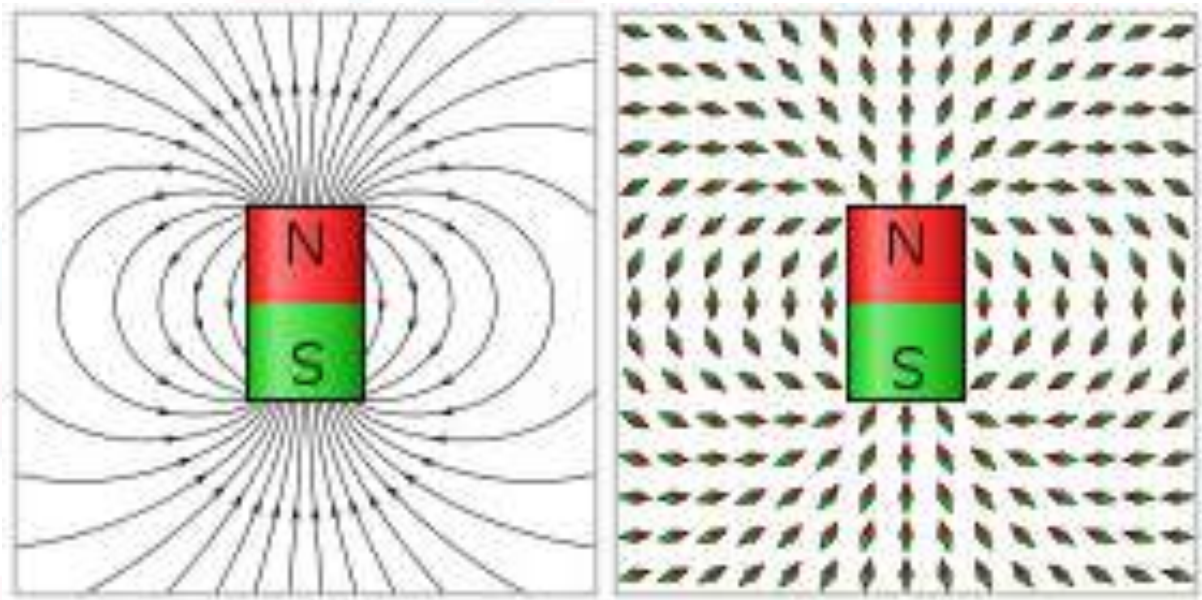
Pontryagin density

Junquera *et al.*, Reviews of Modern Physics 95, 025001 (2023)

Topology: counting and describing singularities

First, a word about "field lines"





https://en.wikipedia.org/wiki/Field_line

Calculating "winding numbers"

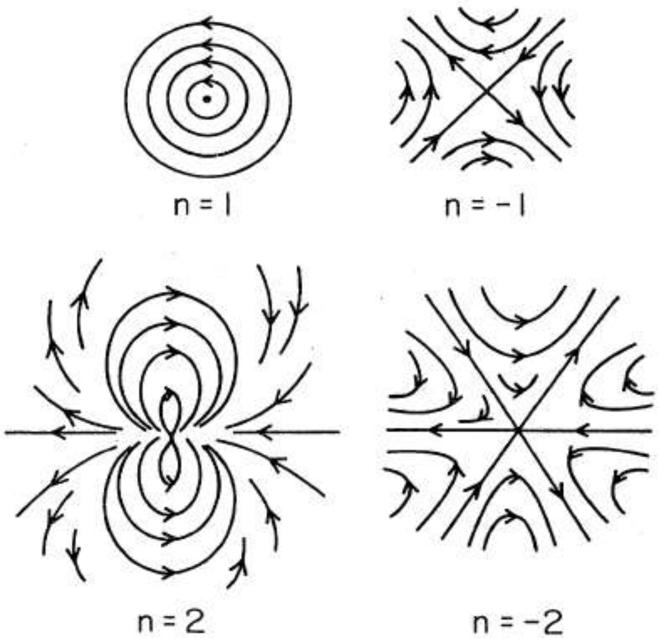


FIG. 1. Point singularities of planar spins in two dimensions with winding numbers ±1 and ±2.

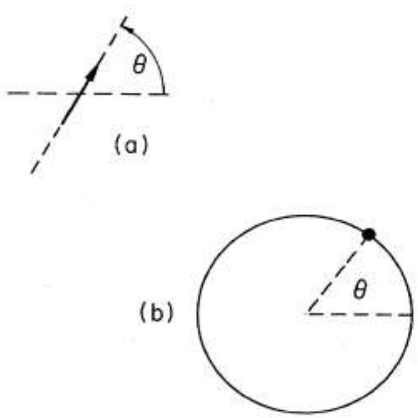
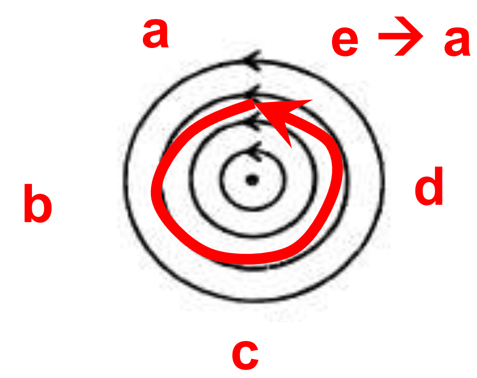
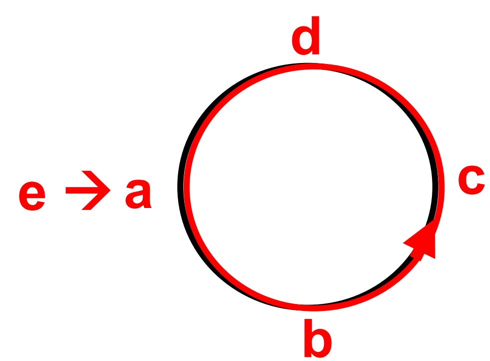


FIG. 3. (a) A planar spin in a given orientation. (b) The representation of that orientation by a point in the order-parameter space.

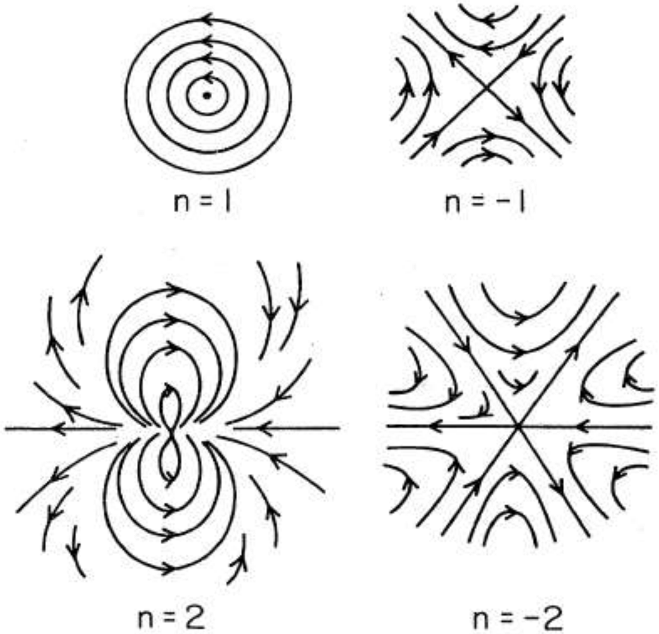




Order-parameter space (S^1)

$$w = \frac{1}{2\pi} \oint d\theta = +1$$

Calculating "winding numbers"



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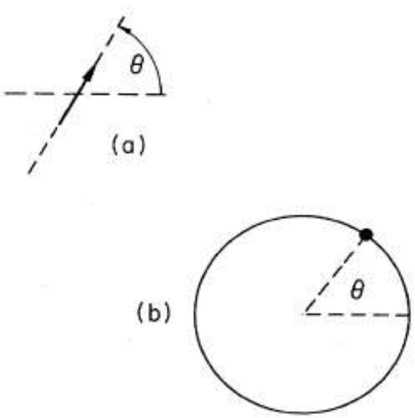

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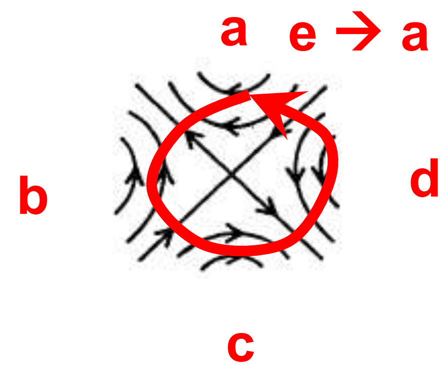
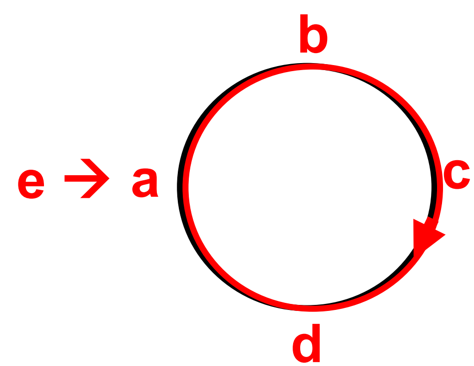
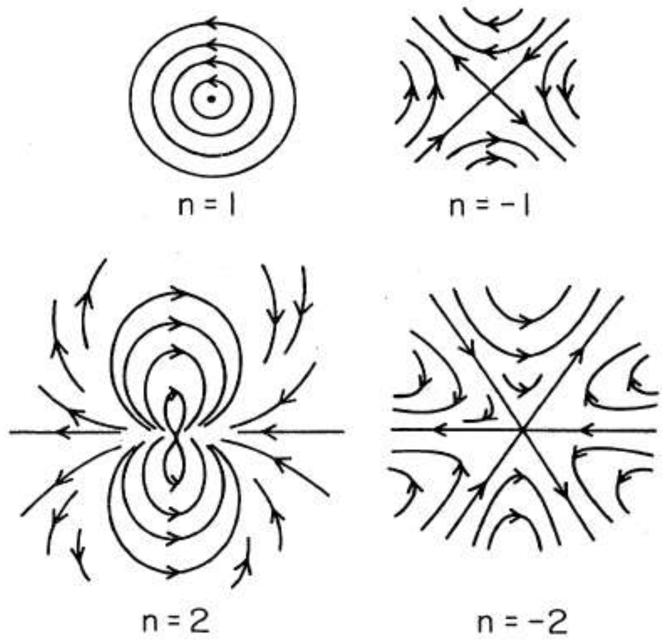




Order-parameter space (S^1)

$$w = \frac{1}{2\pi}\oint d\theta = -1$$

Calculating "winding numbers"



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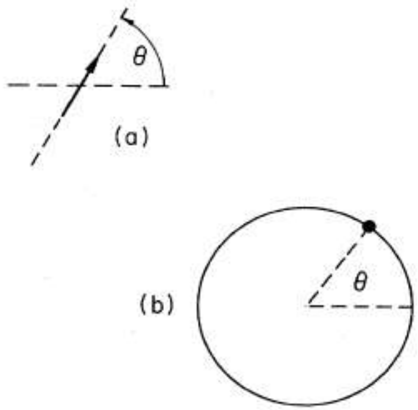

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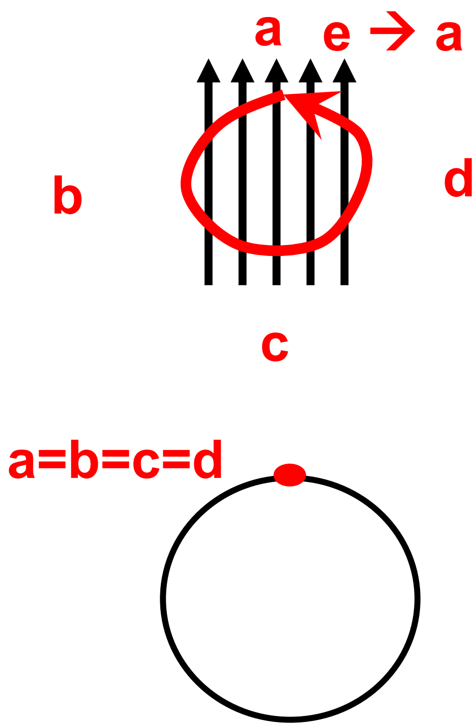
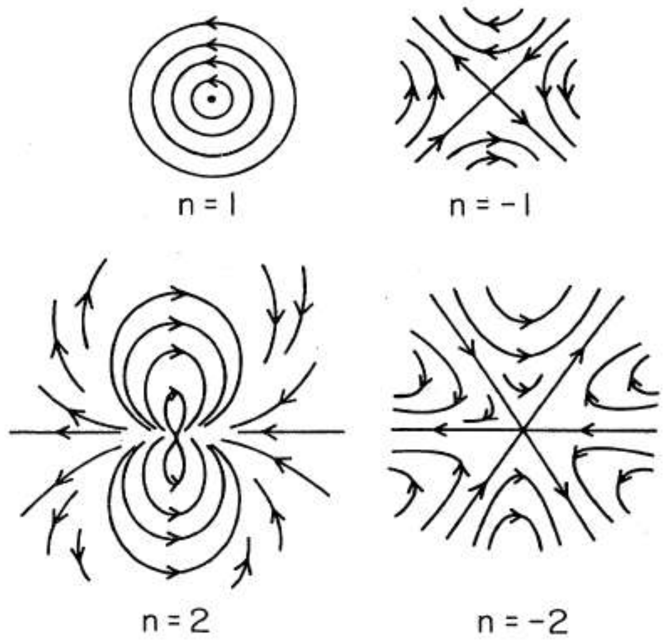


Order-parameter space (S^1)

$$w = \frac{1}{2\pi}\oint d\theta = 0$$

Calculating "winding numbers"



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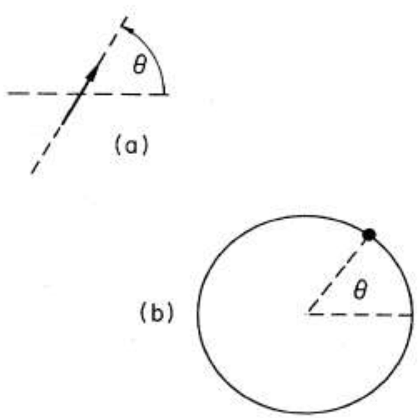

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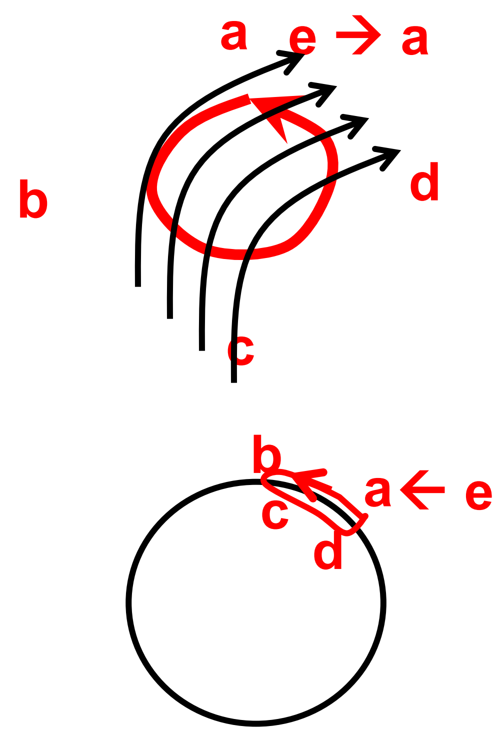


Order-parameter space (S^1)

$$w = \frac{1}{2\pi} \oint d\theta = 0$$

The “defect” is always detected

Provided we have a well-behaved field, we get the same winding number for <u>any</u> path circling the singularity(ies).

→ The field “remembers” the singularity even very far from it.

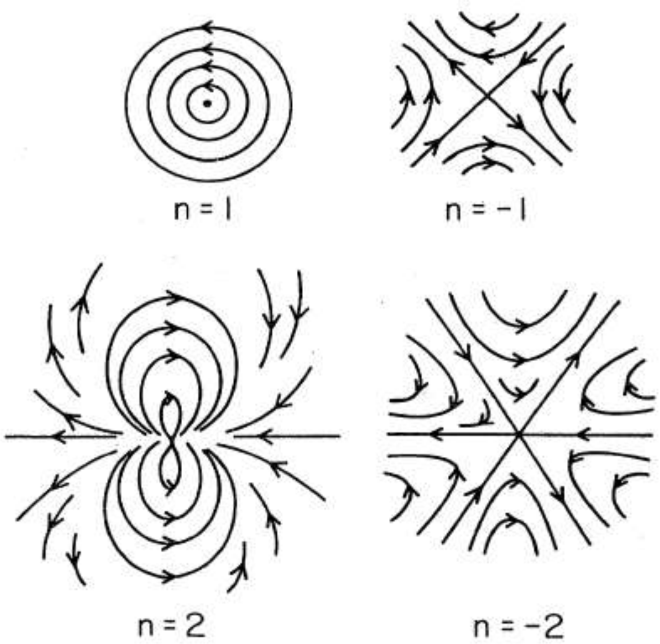
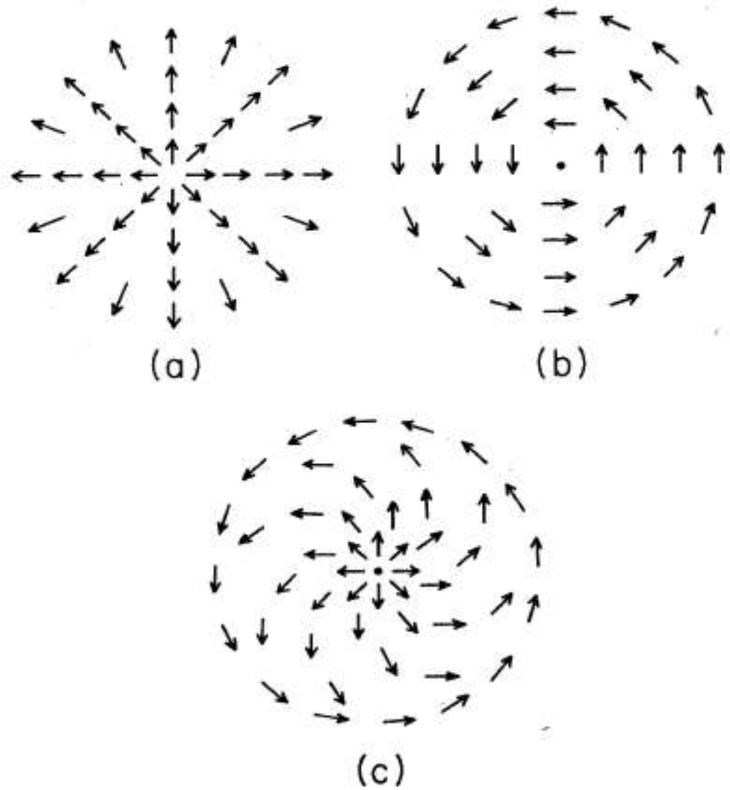
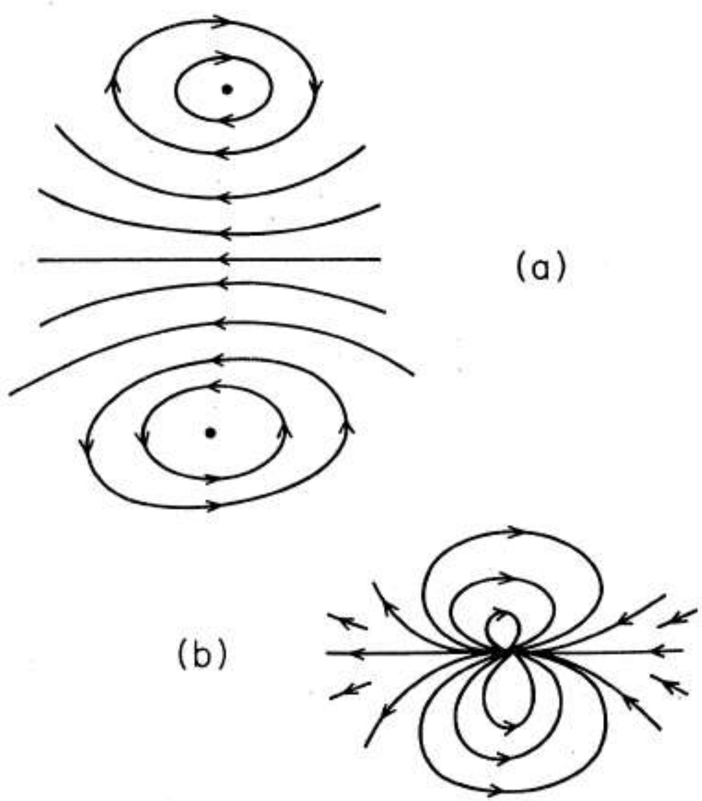


FIG. 1. Point singularities of planar spins in two dimensions with winding numbers ±1 and ±2.

Topological equivalence

Singularities with the same topological number can be deformed into each other while the topological number stays the same at all times → They belong to the same "homotopy class".



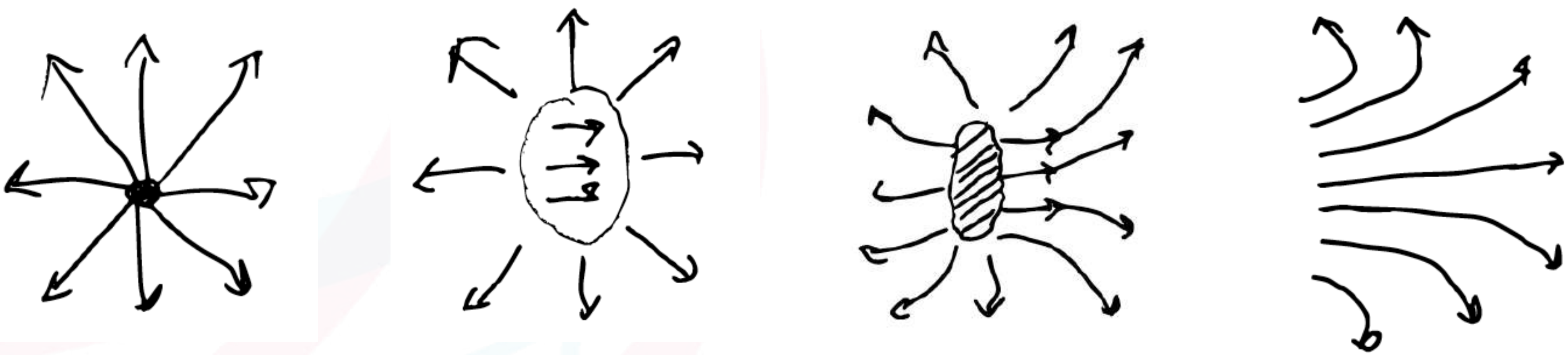


“Topological protection” (I)

In contrast: a singularity Q cannot be removed without altering the field at distances arbitrarily remote from original Q

→ We can safely assume that such a situation will cost some energy

→ **Topological protection !**



w = +1 w = 0

“Topological protection” (II)

Topology does not know anything about interactions / energy

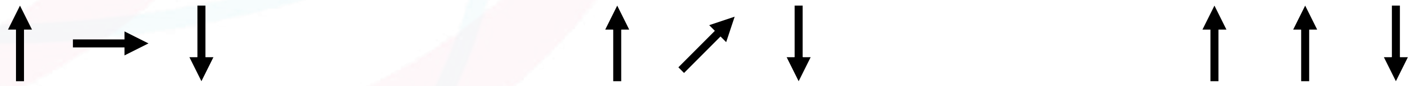
(1) “All topological states are topologically protected” **???**

(2) “… An energy barrier needs to be overcome in order to eliminate a topological state.” ✗

(3) “… This implies that all topological states are minima of the energy” ✗

Not true!

Counter-example: whenever the anisotropy energy ($K > 0, |K| \gg |D|$) dominates, skyrmion-like arrangement is a **local maximum** of energy.



$E \approx -2KS^2$ $E \approx -2.5KS^2$ $E \approx -3KS^2$

"Topological protection" (III)

Where does this "faith" come from?

metastability.[5] However, a topologically stable singularity cannot be obliterated by a mere fluctuation in the local configuration and is, in this sense, physically stable as well.

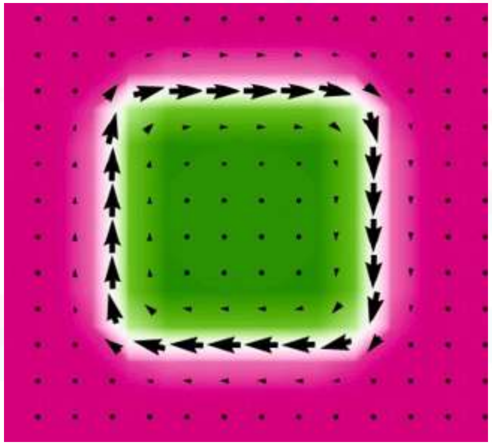
N.D. Mermin, Rev. Mod. Phys. 51, 591 (1979)

A safer (though hardly impressive) statement:

"Provided a (stable) topological singularity exist as a (meta)stable state, we will need to overcome an energy barrier in order to obliterate it and eliminate the topology."

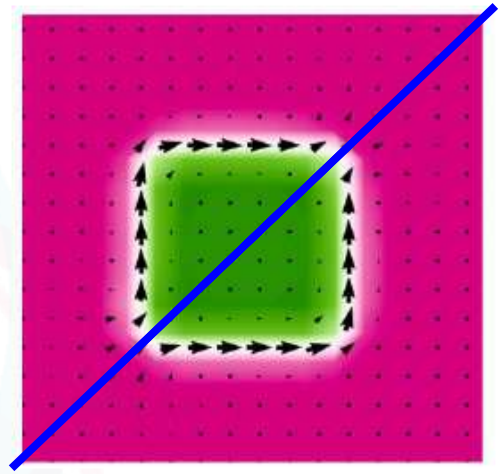
Bottomline: stability is all about the interactions, energy…

Topology → Chirality ?



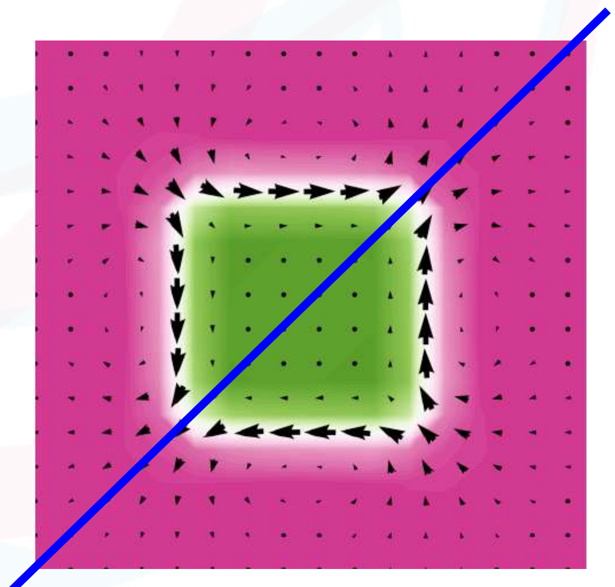
Bloch skyrmion

Topological and chiral



"polar bubble"

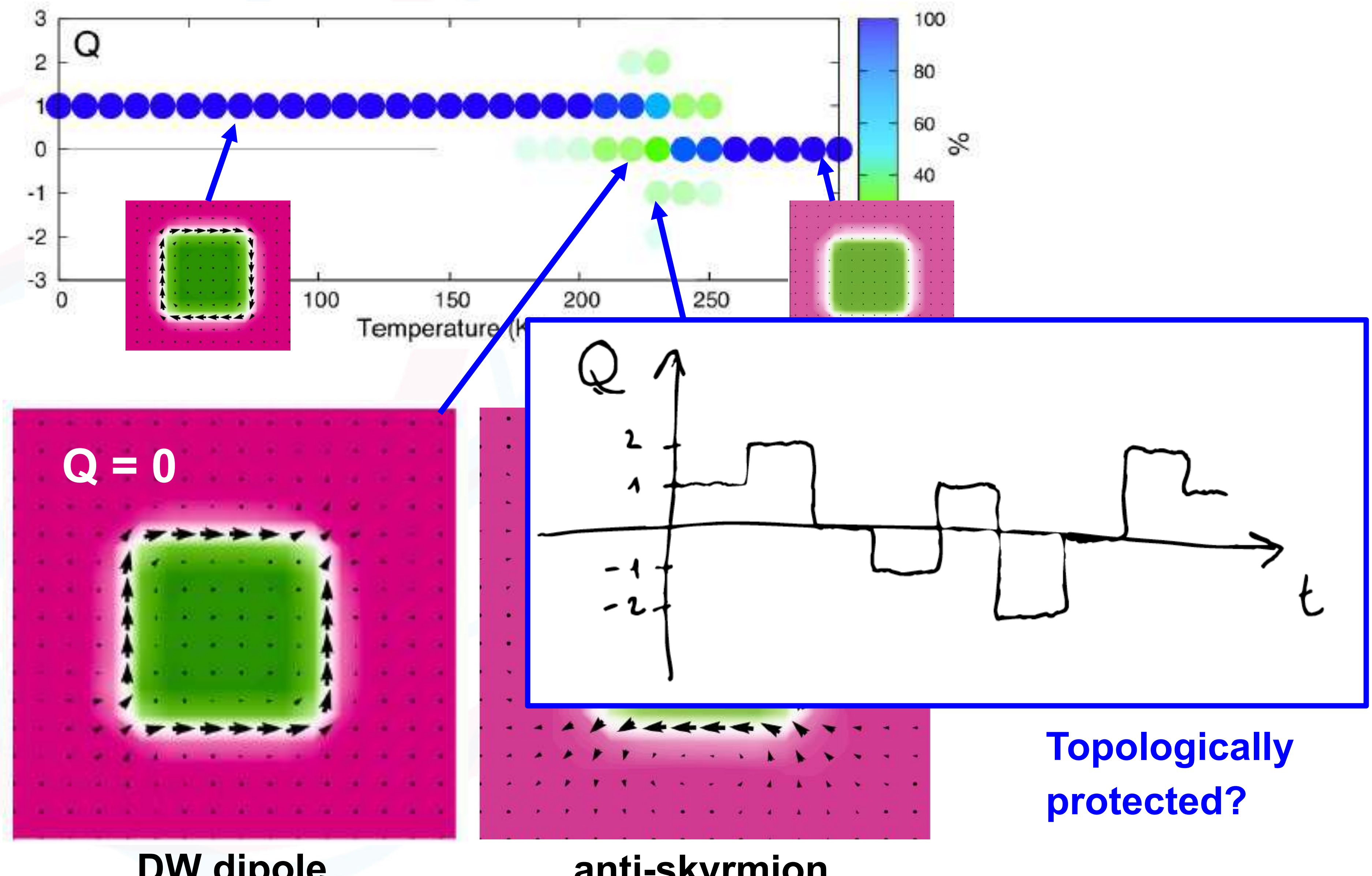
Non-topological and non-chiral



What about this anti-skyrmion?

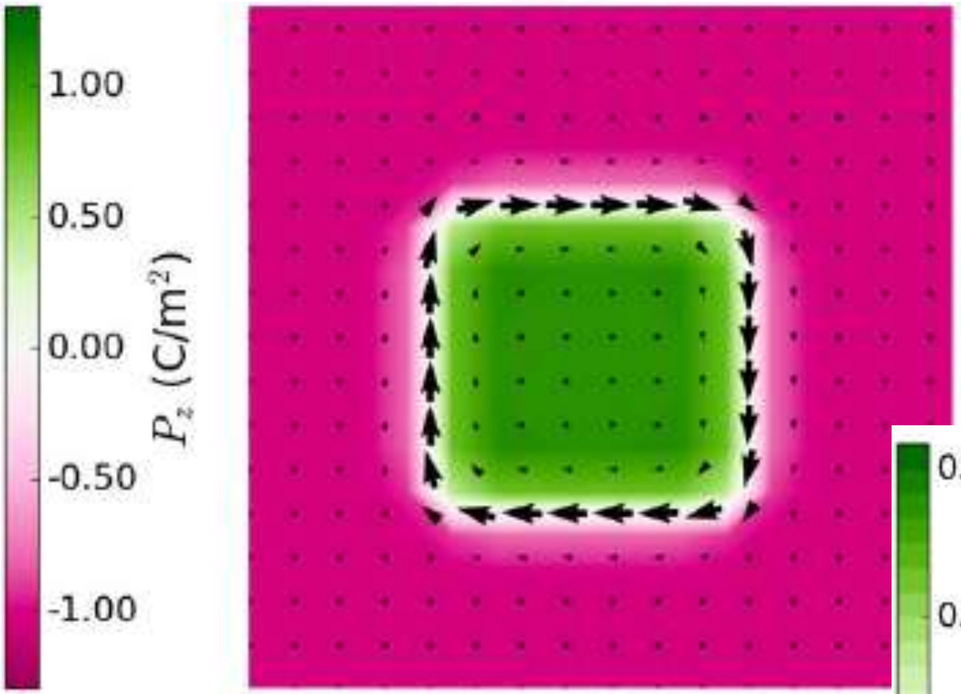
Topological and non-chiral

Specific of ferroelectrics: protection?



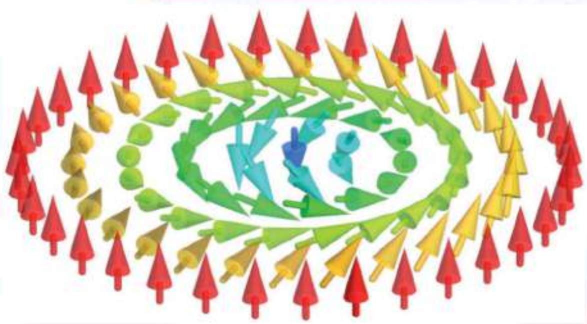
Topologically protected?

Specific of ferroelectrics: quantized?

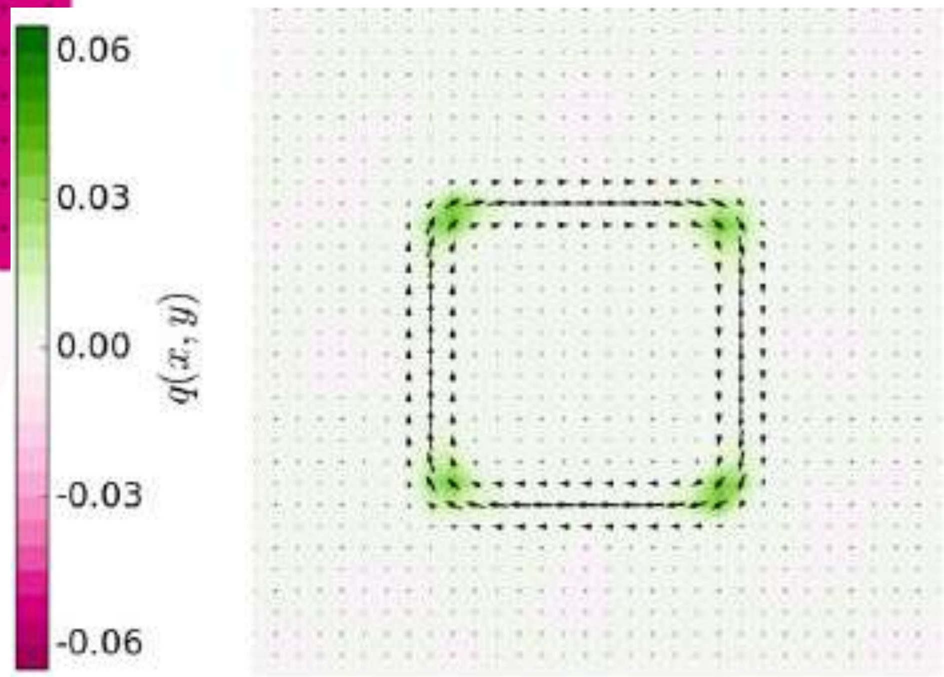


$$Q = \frac{1}{4\pi} \iint d^2\vec{r} \quad \vec{u} \cdot (\partial_x \vec{u} \times \partial_y \vec{u})$$

Pontryagin density *q(x,y)*



Topological charge Q = 1



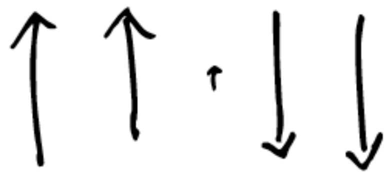
Gonçalves *et al*., Sci. Adv. 5, eaau7023 (2019)

Specific of ferroelectrics: quantized?

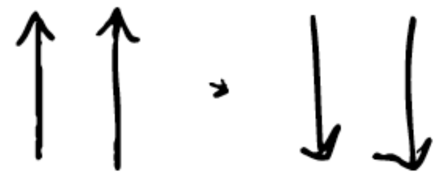
$$Q = \frac{1}{4\pi} \iint d^2\vec{r} \quad \vec{u} \cdot (\partial_x \vec{u} \times \partial_y \vec{u})$$

where $\vec{\boldsymbol{u}}(\boldsymbol{r}) = \dfrac{\vec{\boldsymbol{P}}(\boldsymbol{r})}{|\vec{\boldsymbol{P}}(\boldsymbol{r})|}$

$\vec{\boldsymbol{P}}(\boldsymbol{r})$

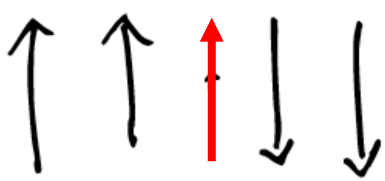


surely trivial

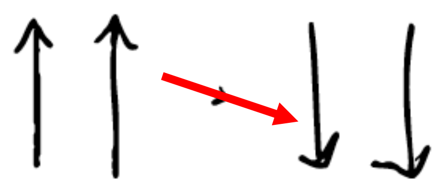


surely trivial

$\vec{\boldsymbol{n}}(\boldsymbol{r})$

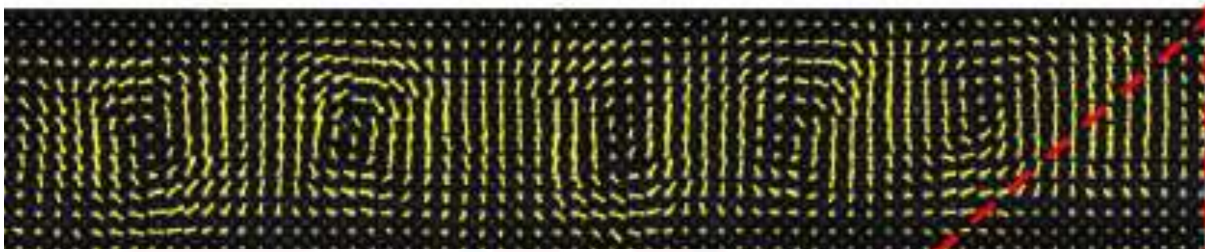


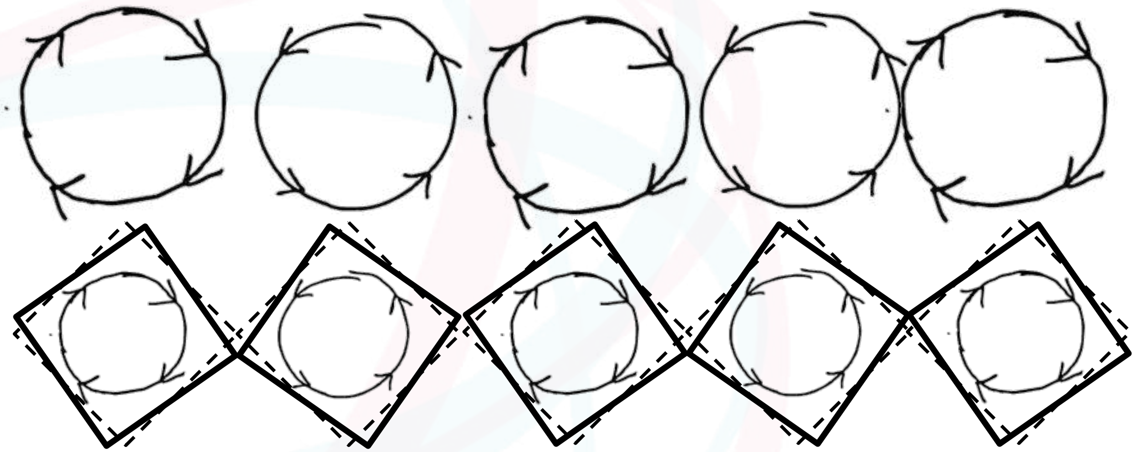
still trivial

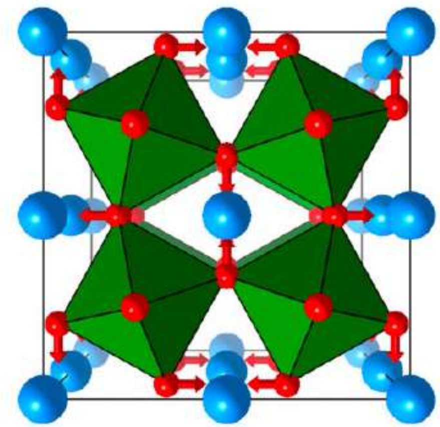


topological ??

Is there a DMI in perovskite oxides?





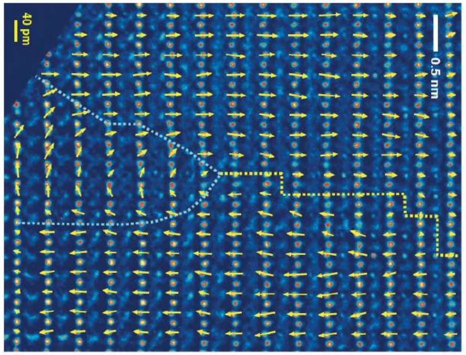


Can’t get more exciting than this !!

Non-collinear electric dipoles

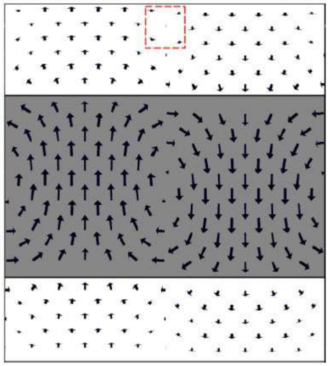
2004

2011

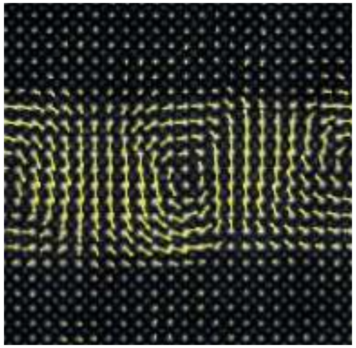


Vortex-like domain walls

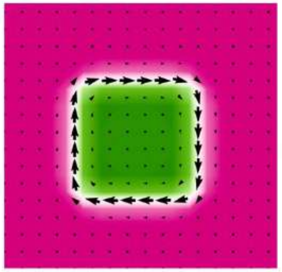
2012



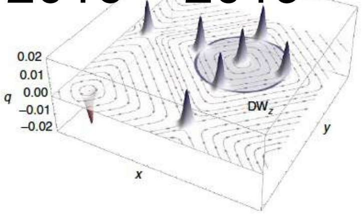
2016

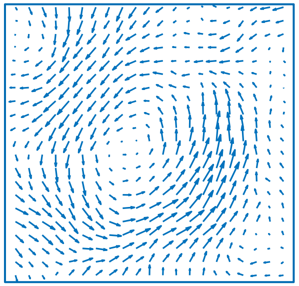


Electric skyrmion bubble



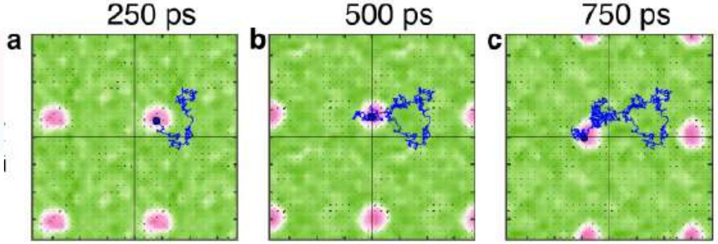
2016 – 2019





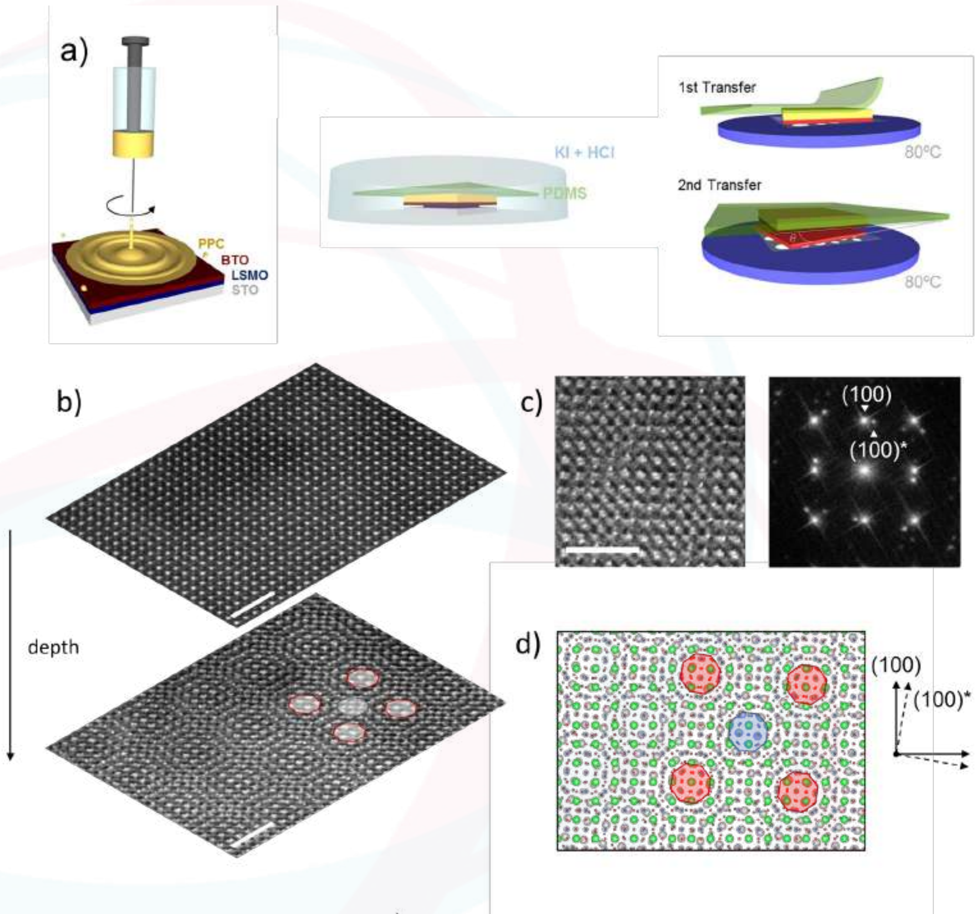
2019

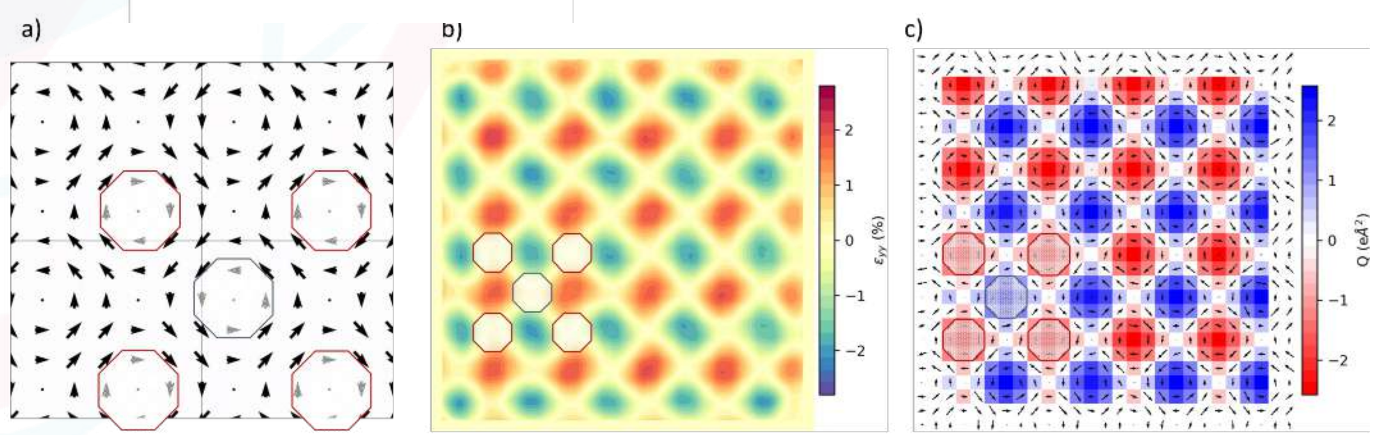
Electric bubble quasiparticle



2023

….





Sánchez-Santolino et al.,

https://arxiv.org/abs/2301.04438

Thank you
for your attention!