



# UCL

**LCN**  
LONDON CENTRE FOR  
NANOTECHNOLOGY

## Introduction to Ferroelectrics

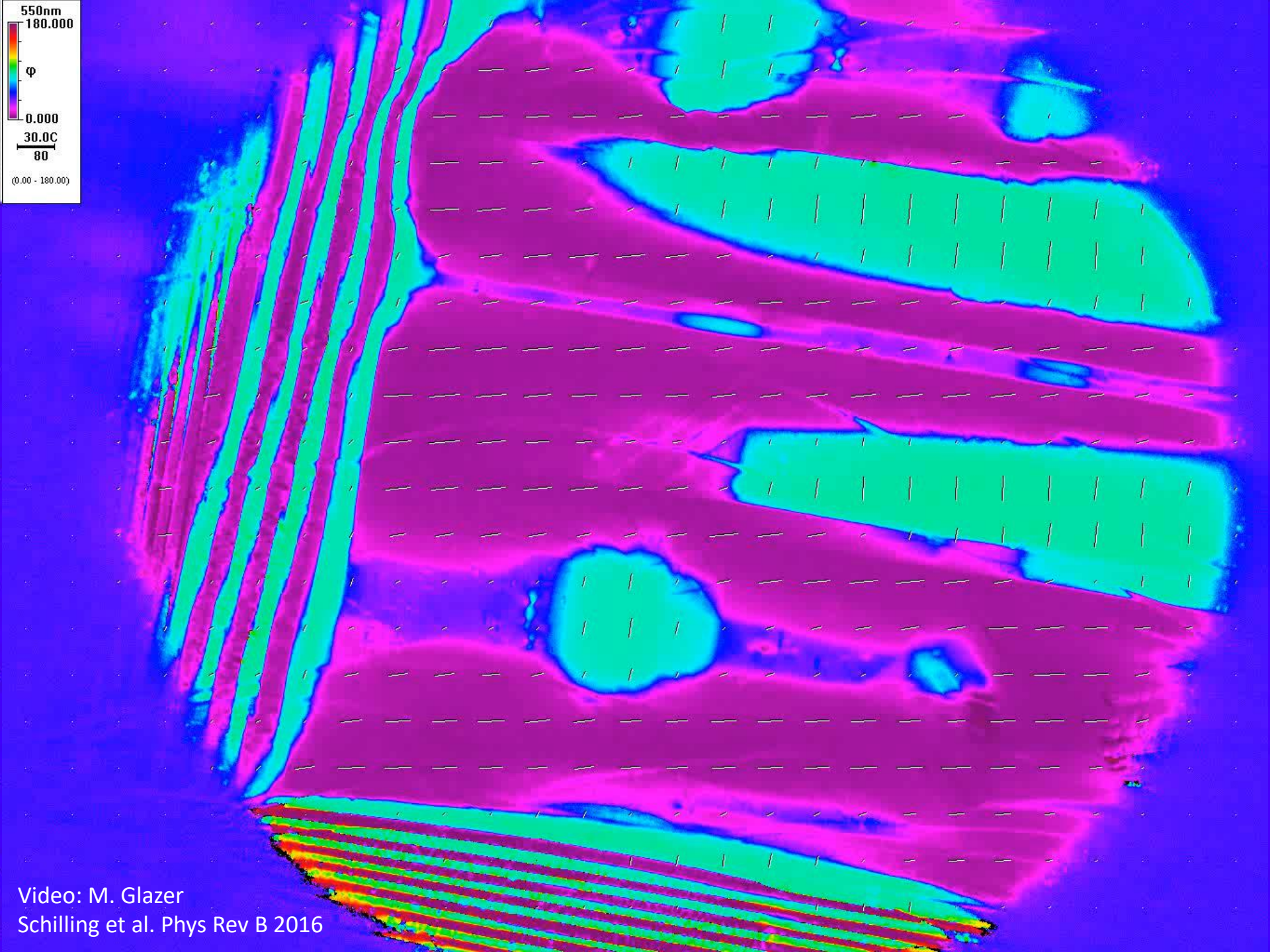
Pavlo Zubko

University College London

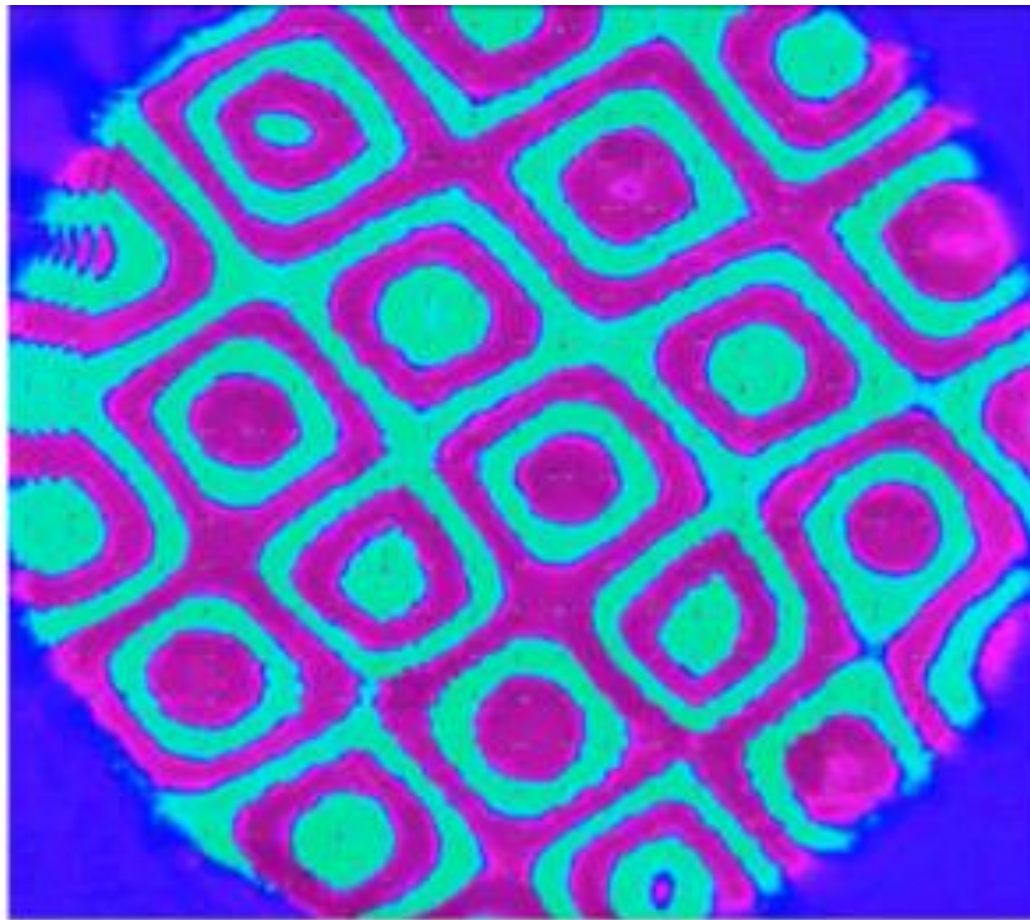
ISOE – Cargèse – 2023

Image: Forsbergh, Phys. Rev. 1949





Video: M. Glazer  
Schilling et al. Phys Rev B 2016



Schilling et al. Phys Rev B 2016

Forsbergh (Phys. Rev. 1949)



PIEZOELECTRIC AND ALLIED PHENOMENA IN ROCHELLE SALT.

BY JOSEPH VALASEK.

(AKA potassium sodium tartrate tetrahydrate,  $\text{KNaC}_4\text{H}_4\text{O}_6 \cdot 4\text{H}_2\text{O}$ )



Joseph Valasek in 1922.

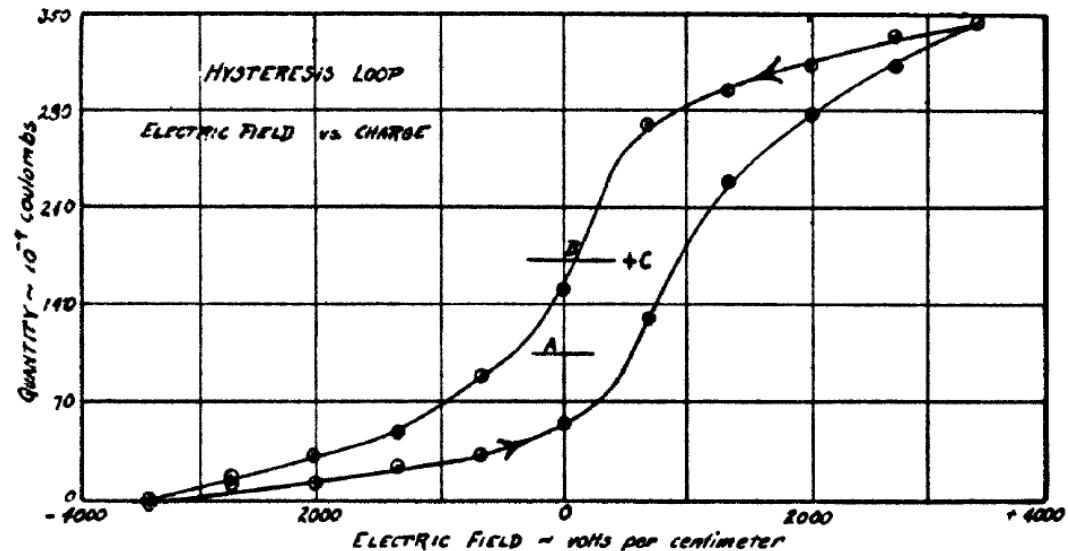
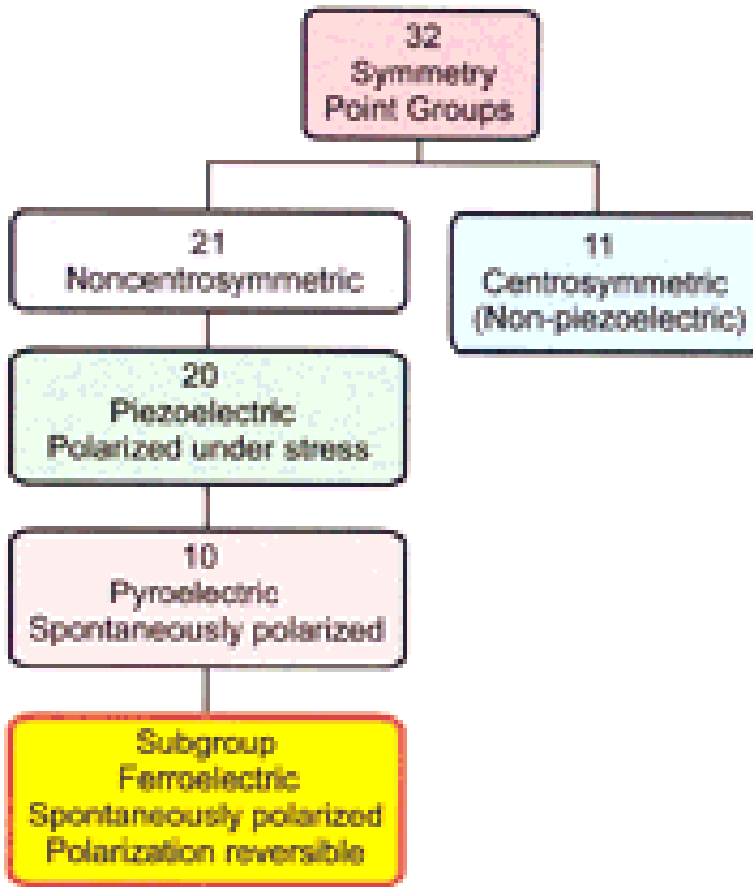


Fig. 4.

Physical Review 1921

Ferroelectrics named after ferromagnets (not because they contain Fe...but there are some, e.g.  $\text{BiFeO}_3$ )



Haertling J. Am. Cer. Soc. 1999

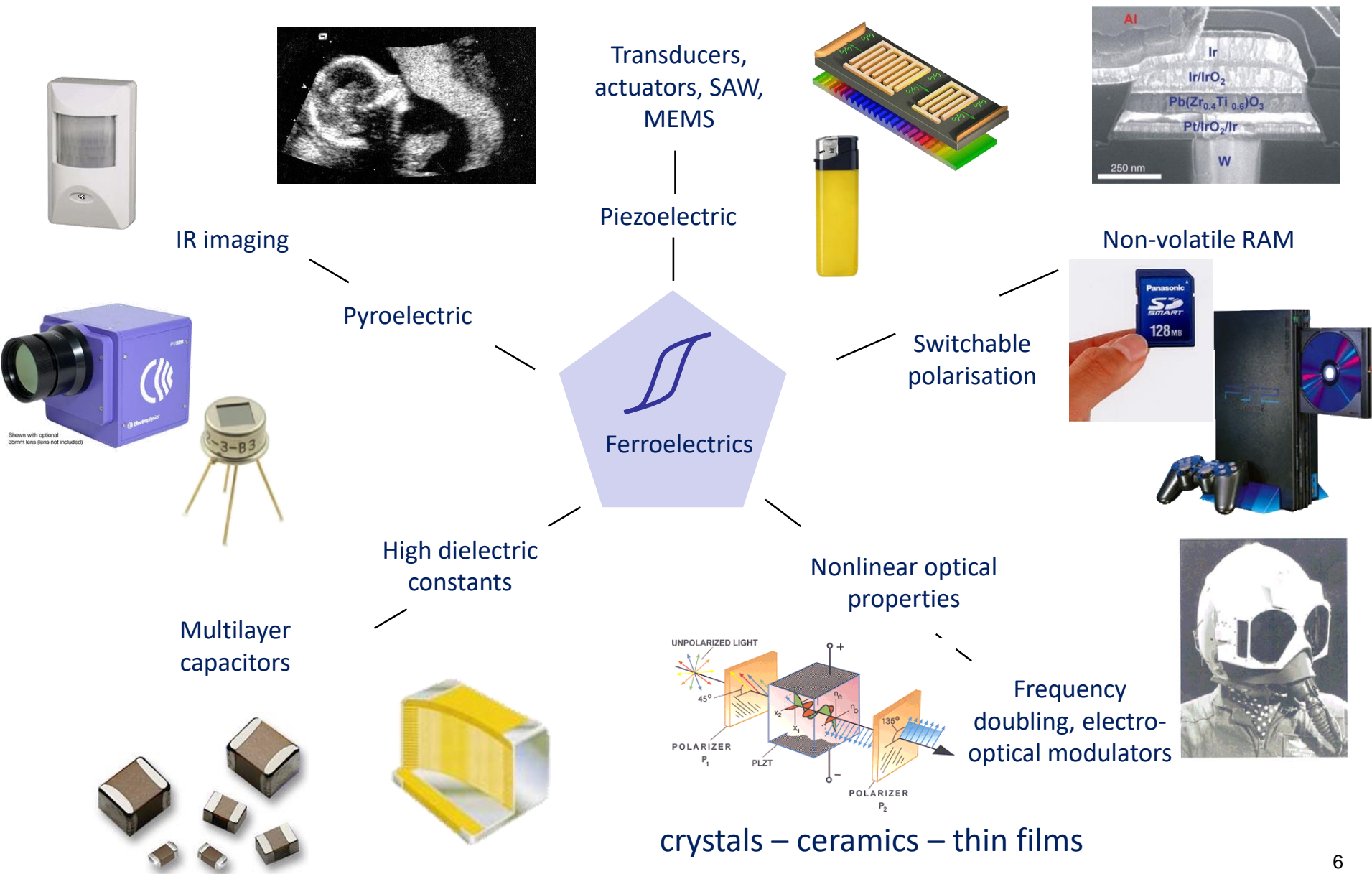
Ferroelectrics:

- 1) Possess spontaneous polarisation  
(restricted by symmetry to pyroelectric point groups)

+

- 2) Polarisation must be switchable by applied field (below breakdown field)  
(practical restriction)

- All ferroelectrics are piezoelectric & pyroelectric



## Rochelle salt

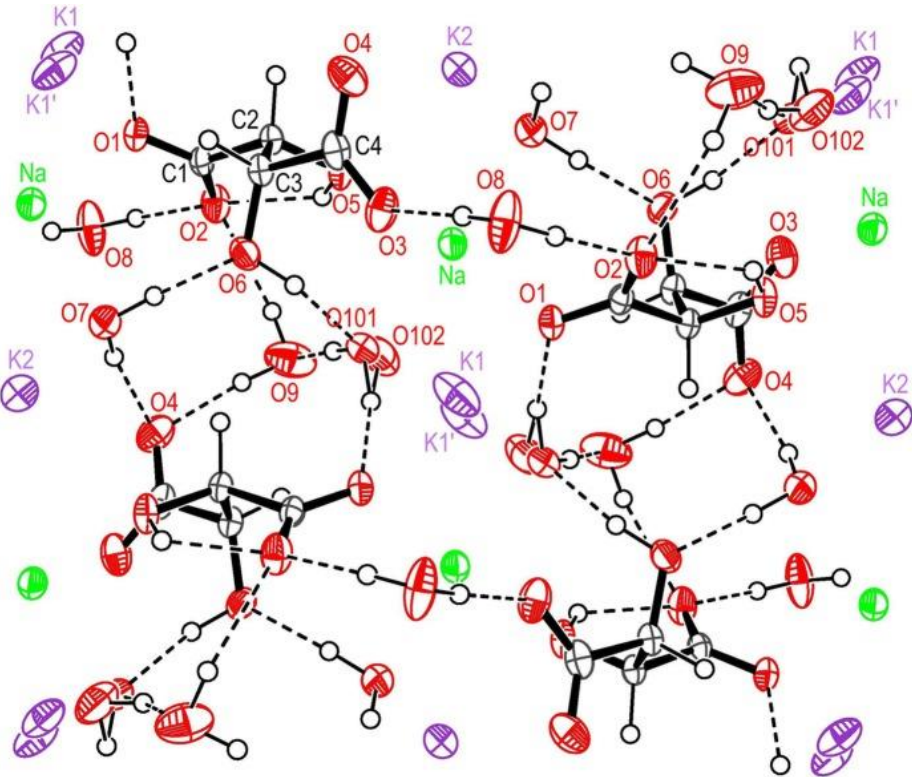


Image: Mo et al. IUCrJ 2015

Wide range of crystal structures from very complex to very simple

- $\text{KH}_2\text{PO}_4$  (KDP),
- $(\text{NH}_2\text{CH}_2\text{COOH})_3\text{H}_2\text{SO}_4$  (TGS)
- P(VDF-TrFE)
- $\text{NaNO}_2$
- HCl, HBr
- and many, MANY others

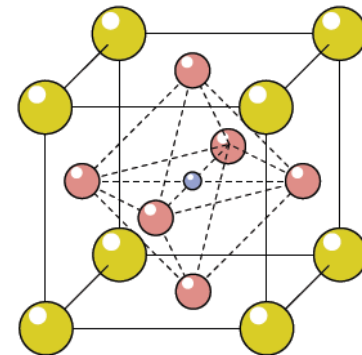
## Oxides

### Perovskites

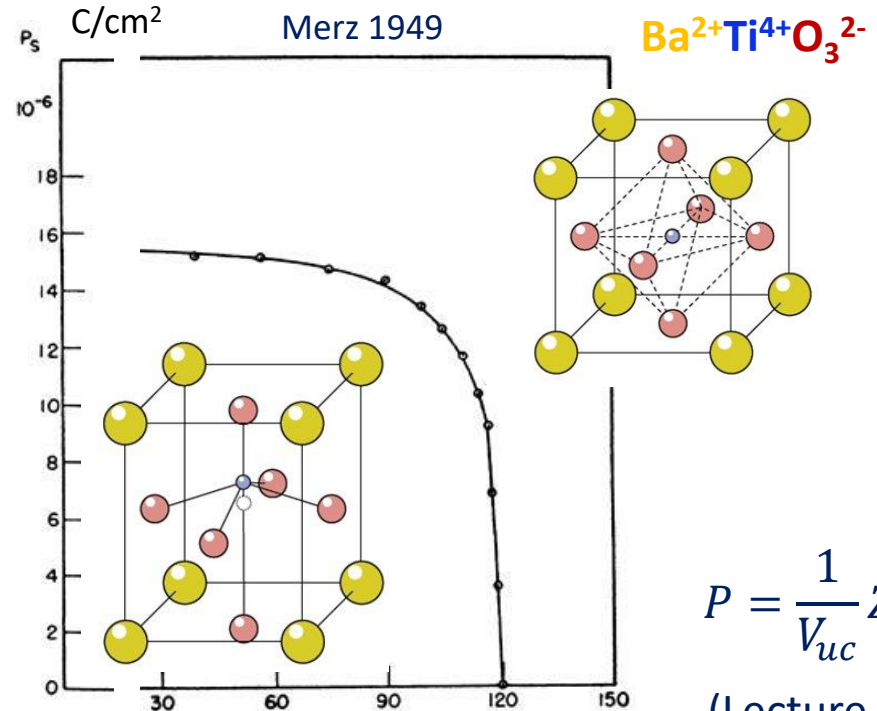
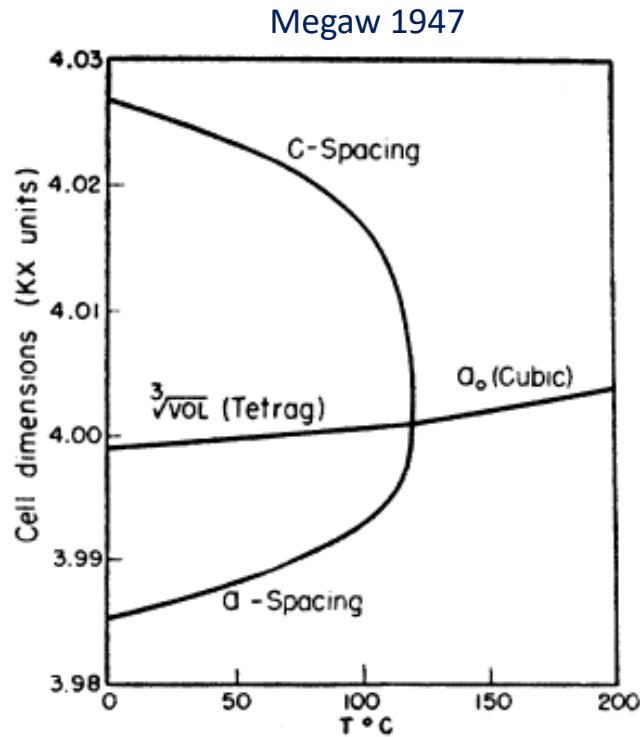
- $\text{BaTiO}_3$ ,  $\text{PbTiO}_3$ ,  $\text{KNbO}_3$ ,  $\text{BiFeO}_3$ ,  $\text{Pb}(\text{Zr,Ti})\text{O}_3$  (PZT)...

### Other oxides:

- $\text{LiNbO}_3$ ,  $\text{LiTaO}_3$ ,  $\text{SrBi}_2\text{Ta}_2\text{O}_9$  (SBT),  $\text{Bi}_4\text{Ti}_3\text{O}_{12}$ ,  $\text{HfO}_2$  ...



$\text{BaTiO}_3$   
(1944)



$$P = \frac{1}{V_{uc}} Z^* u$$

(Lecture by Karin Rabe!)

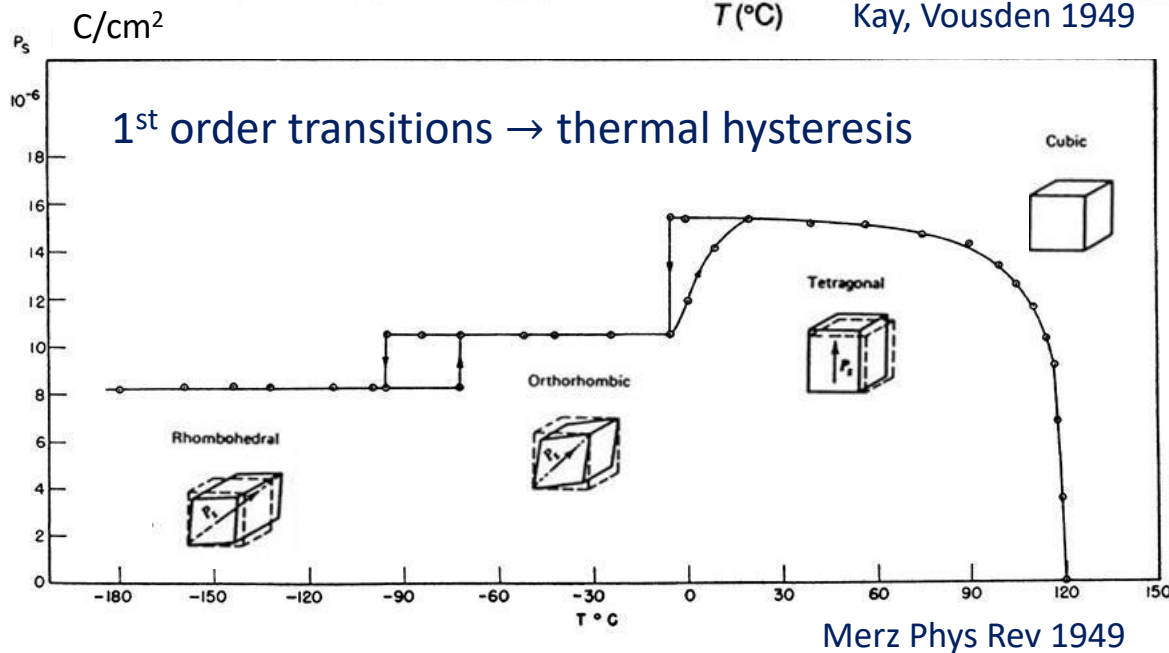
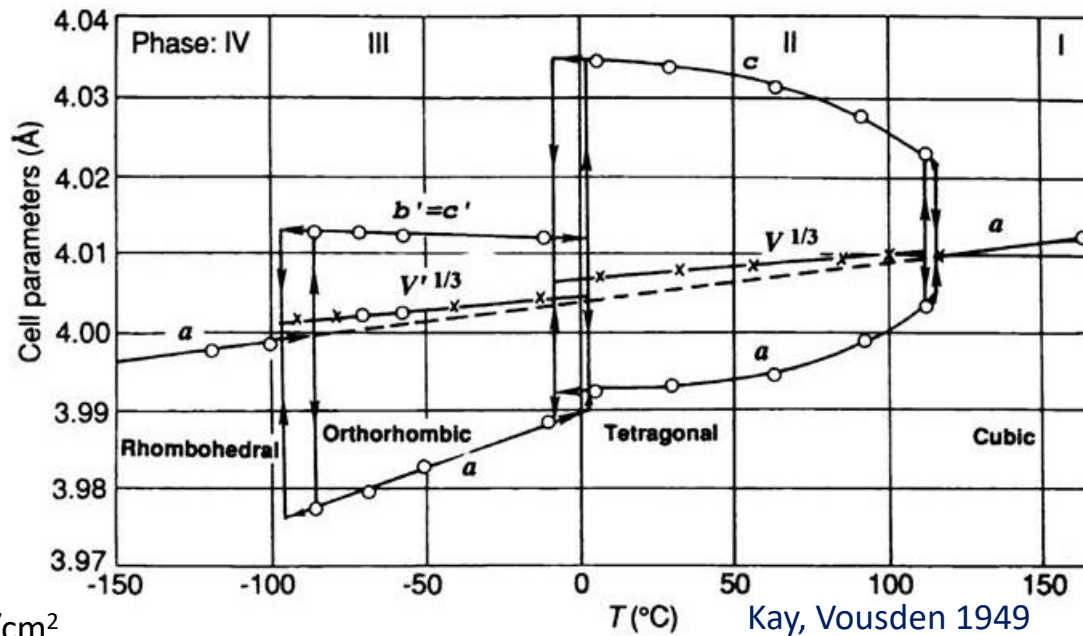
Phase transition:

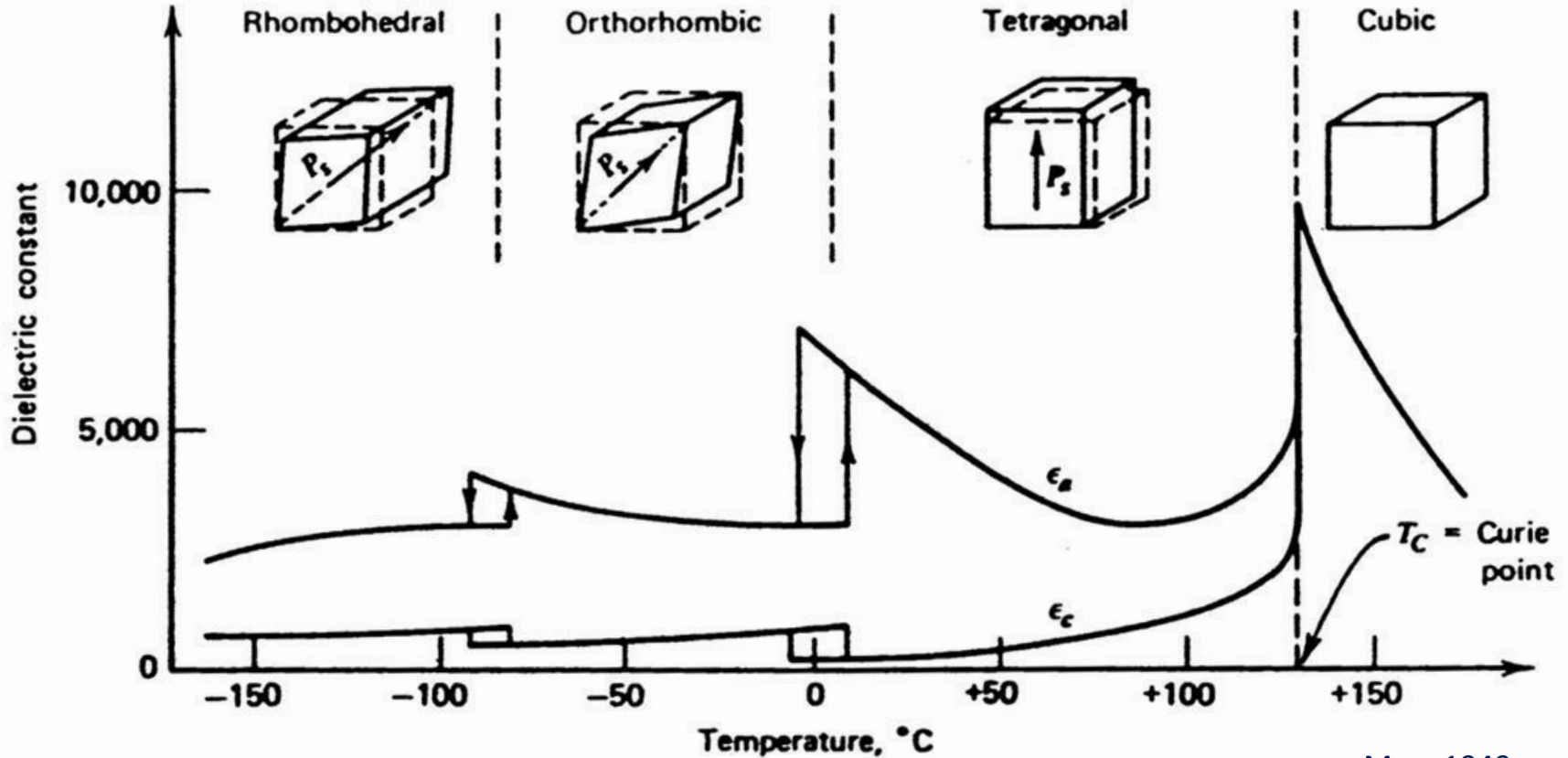
$T > T_C$  - cubic, centrosymmetric

$T < T_C$  - tetragonal, polar

Cooperative alignment of dipoles → macroscopic polarisation







Merz 1949

Huge dielectric constants! Especially near  $T_C$

BaTiO<sub>3</sub>-based dielectrics used in multilayer capacitors

## Landau theory

- based on symmetry considerations
- provides no microscopic insight...but...
- links different macroscopic properties through thermodynamics

Landau ansatz: expand free energy in powers of order parameter(s).

Primary OP =  $P$  (proper ferroelectrics)

$$F = \frac{\alpha}{2}P^2 + \frac{\beta}{4}P^4 + \frac{\gamma}{6}P^6 + \dots - EP$$

Allowed terms constrained by symmetry (free energy must describe ferro- and para- phase)

Landau – Devonshire



Simplest potential describing a phase transition

$$F = \frac{\alpha}{2}P^2 + \frac{\beta}{4}P^4 - EP$$

Assume:

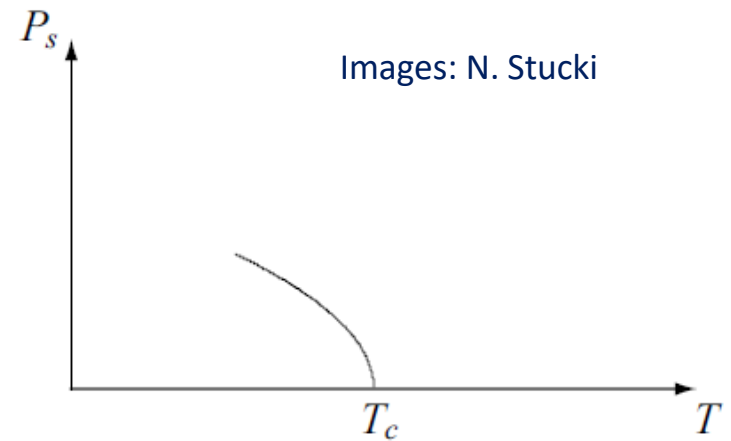
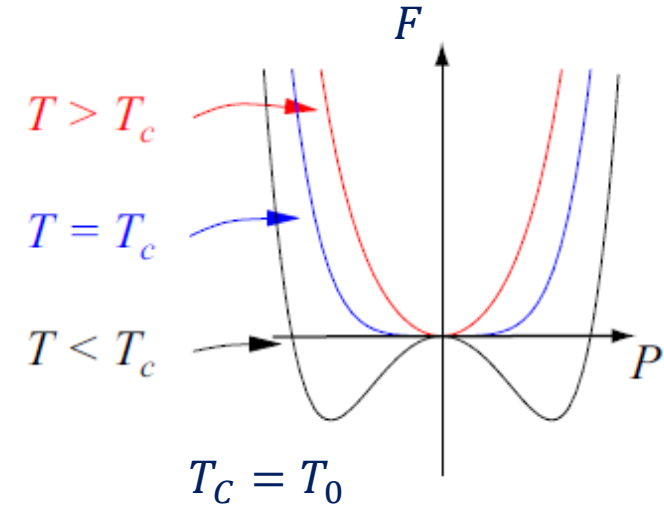
- $\alpha = \alpha_0(T - T_0)$
- $\beta > 0$  & temp. independent (last term in expansion must be positive for stability)

In zero field, equilibrium when  $\frac{\partial F}{\partial P} = 0$

$$\frac{\partial F}{\partial P} = \alpha P + \beta P^3 = 0$$

→ spontaneous polarisation  $P_s$

$$P_s^2 = -\frac{\alpha}{\beta} = \frac{\alpha_0(T_0 - T)}{\beta}$$



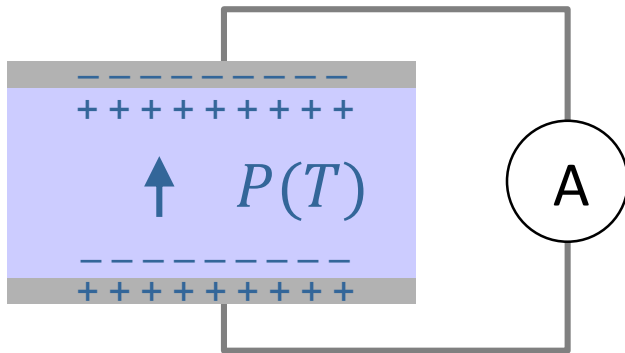
2<sup>nd</sup> order PT – OP changes continuously

Pyroelectricity

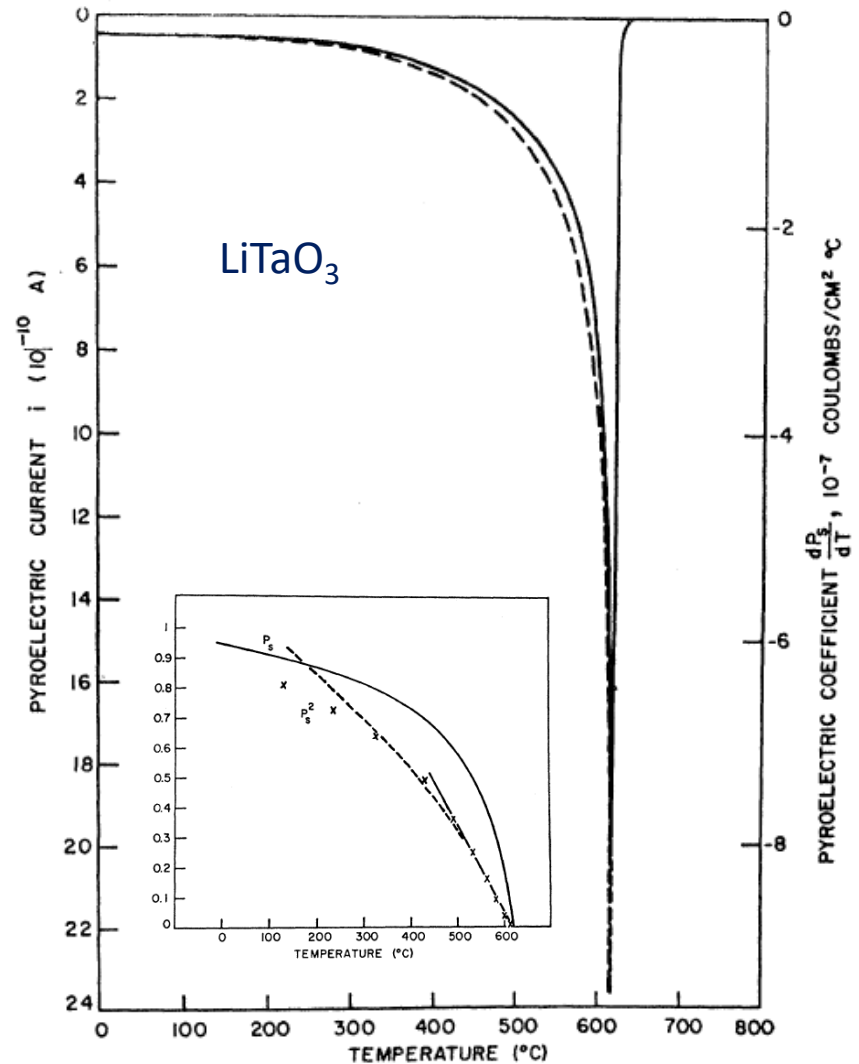
$$\pi = \frac{dP}{dT}$$

Change  $T$ , measure short circuit pyroelectric current

$$j = \frac{dD}{dt} = \frac{dP}{dt}$$



$$D = P + \epsilon_0 E = P \quad (E = 0)$$



Glass, Phys. Rev. 1968

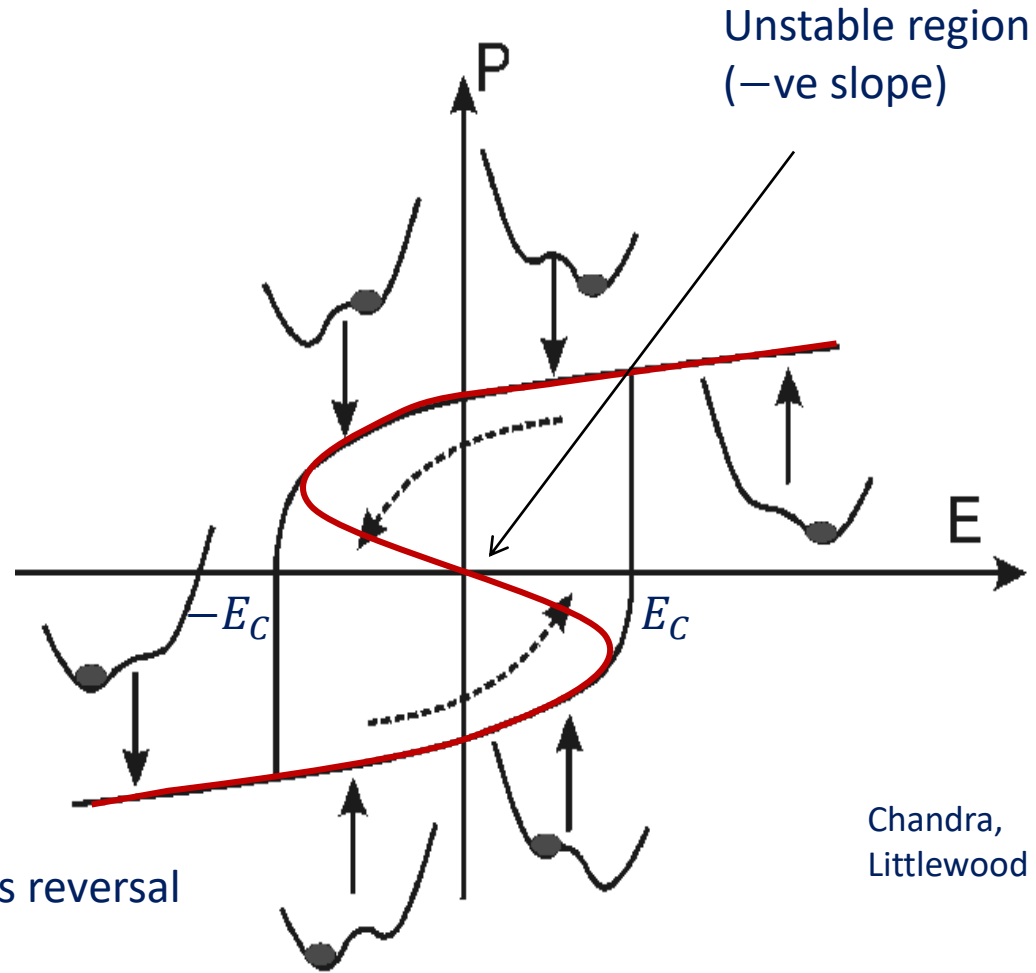
$$F = \frac{\alpha}{2}P^2 + \frac{\beta}{4}P^4 - EP$$

In electric field,  $\frac{\partial F}{\partial P} = 0$  gives

$$\alpha P + \beta P^3 - E = 0$$

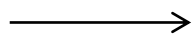
→ Ferroelectric hysteresis

→ Intrinsic coercive field  $E_C$



Chandra,  
Littlewood

Intrinsic coercive field (homogeneous reversal of polarisation)  $\gg$  experimental  $E_C$

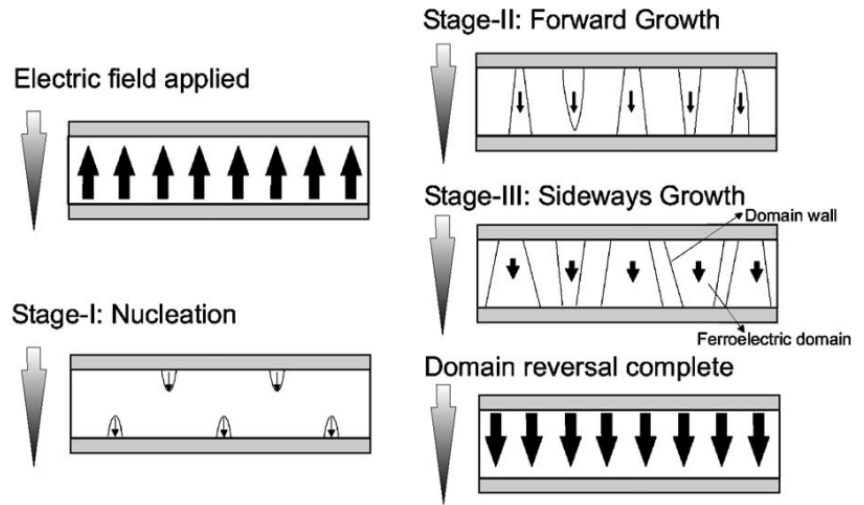
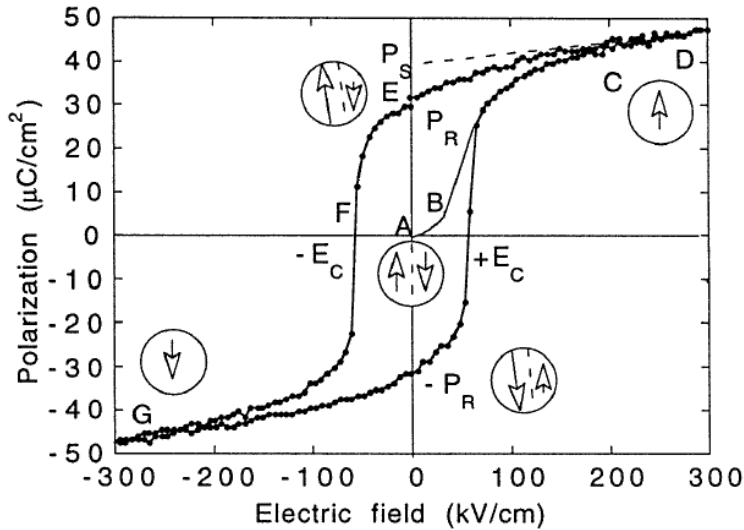


In practice switching is domain mediated and the intrinsic coercive field is not observed



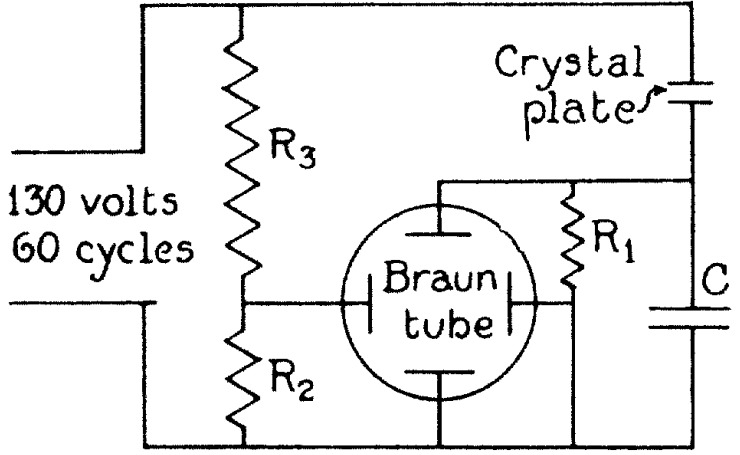
Switching proceeds via domain nucleation and growth

Damjanovic, Rep. Prog. Phys. 1998



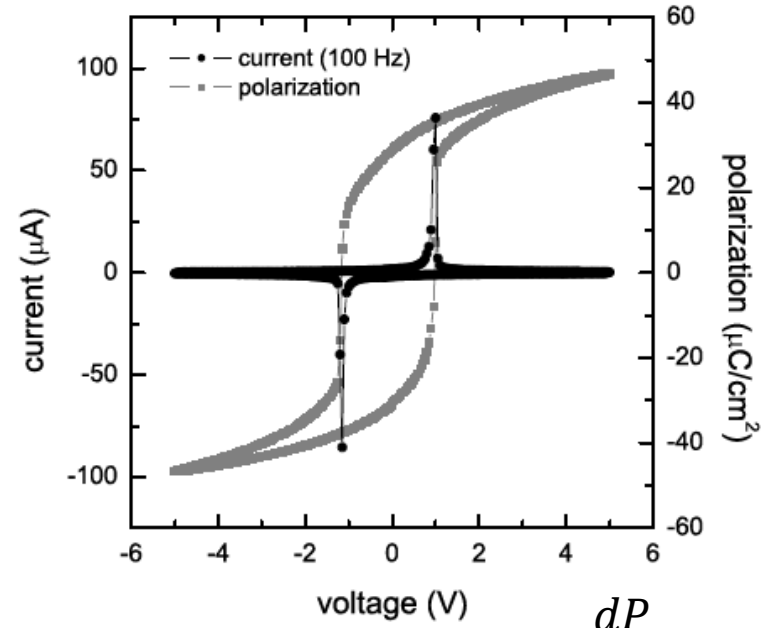
Dawber et al. Rev. Mod. Phys. (2005)

Switching is usually inhomogeneous (nucleation at predetermined sites)



Sawyer-Tower method

Sawyer, Tower, Phys. Rev. 35, 269 (1930)



Commercial testers

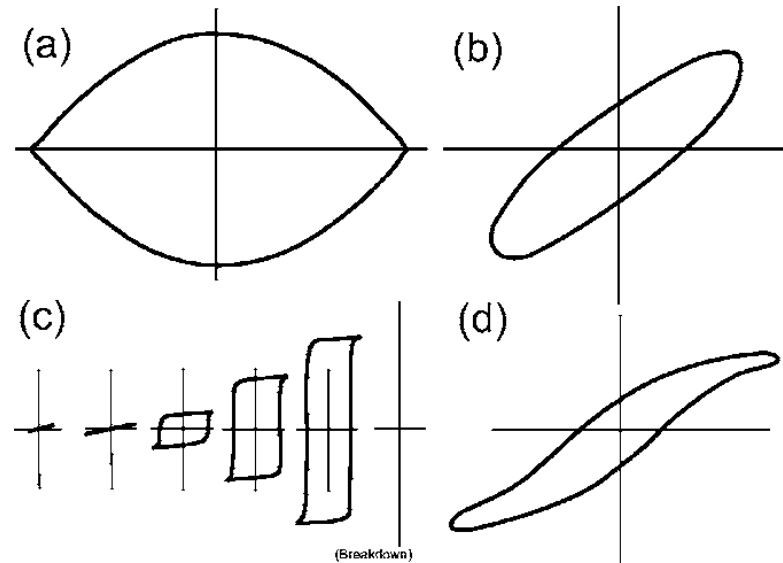
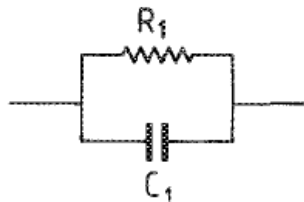
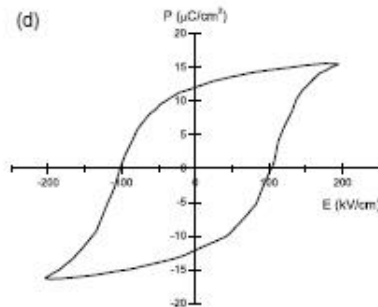
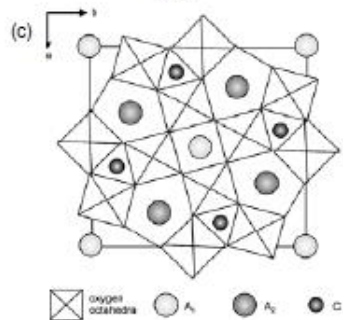
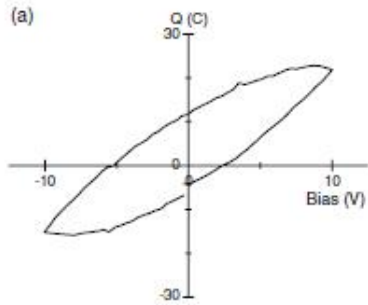


**VIEWPOINT**

# Ferroelectrics go bananas

$$j = \frac{dP}{dt} + \sigma_{dc}E$$

J F Scott



Common hysteresis artifacts:

- (a) dead short
- (b) linear lossy dielectric,
- (c) saturated amplifier
- (d) nonlinear lossy dielectric.

Dawber et al. Rev. Mod. Phys. (2005)



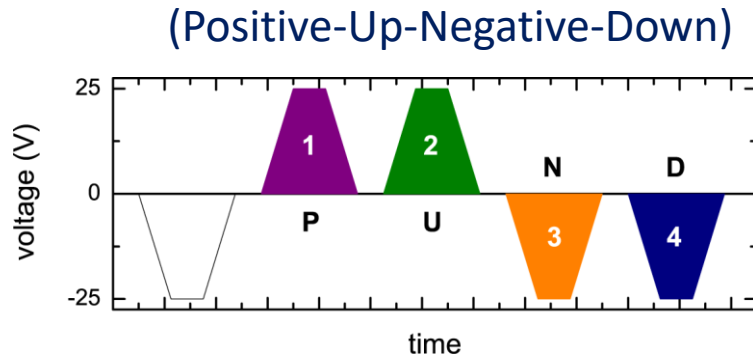


Image: N. Stucki

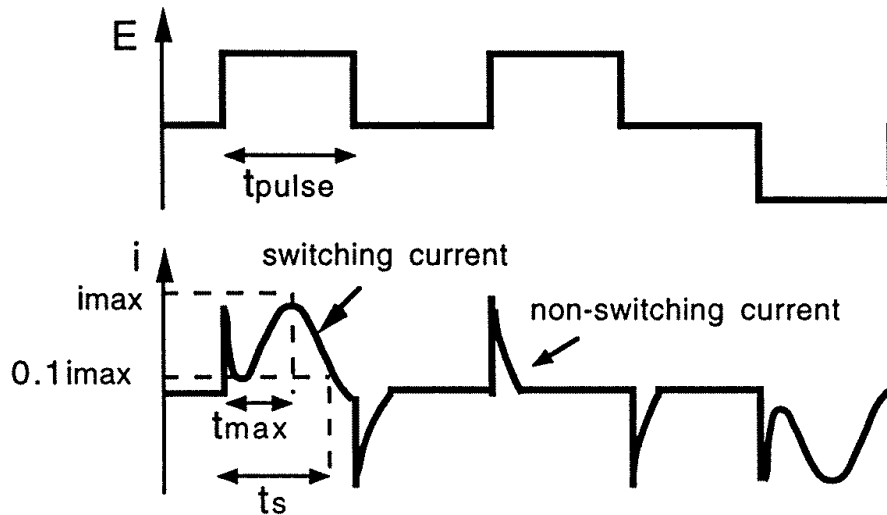
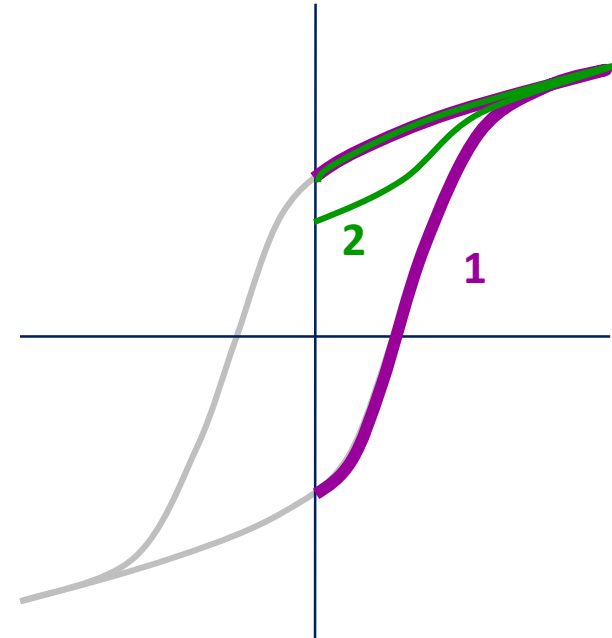
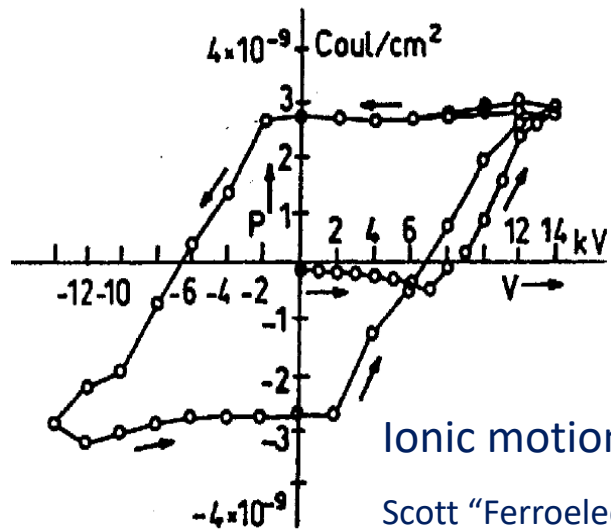


Image: Damjanovic, Rep. Prog. Phys. 1998

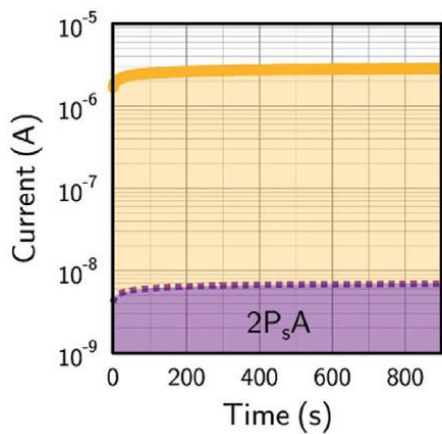
- Look at switching current
- Frequency dependence
- PUND

Note: only measure **changes** in polarisation



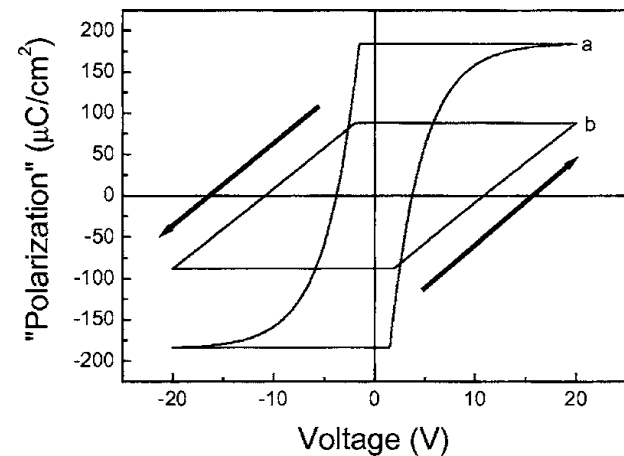
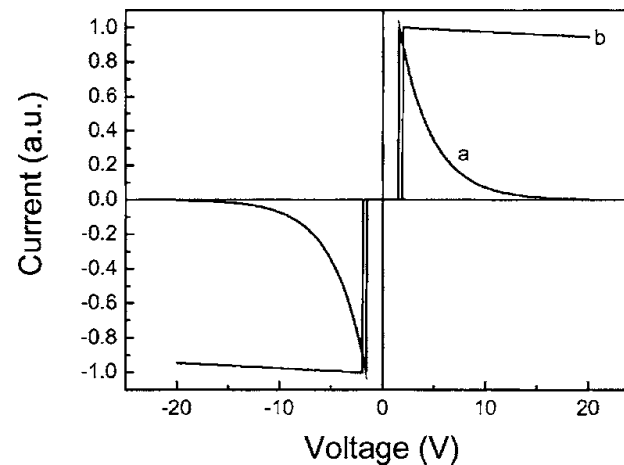
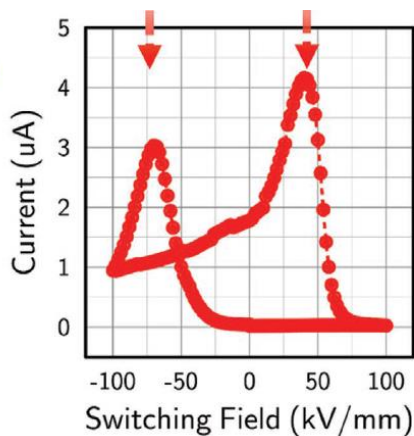
Ionic motion (electrets)

Scott "Ferroelectric Memories"



Domain wall conductivity

McClusky et al. Adv. Func. Mater. 2020



Emission from traps, back to back Schottky diodes

Pintilie & Alexe, APL 2005

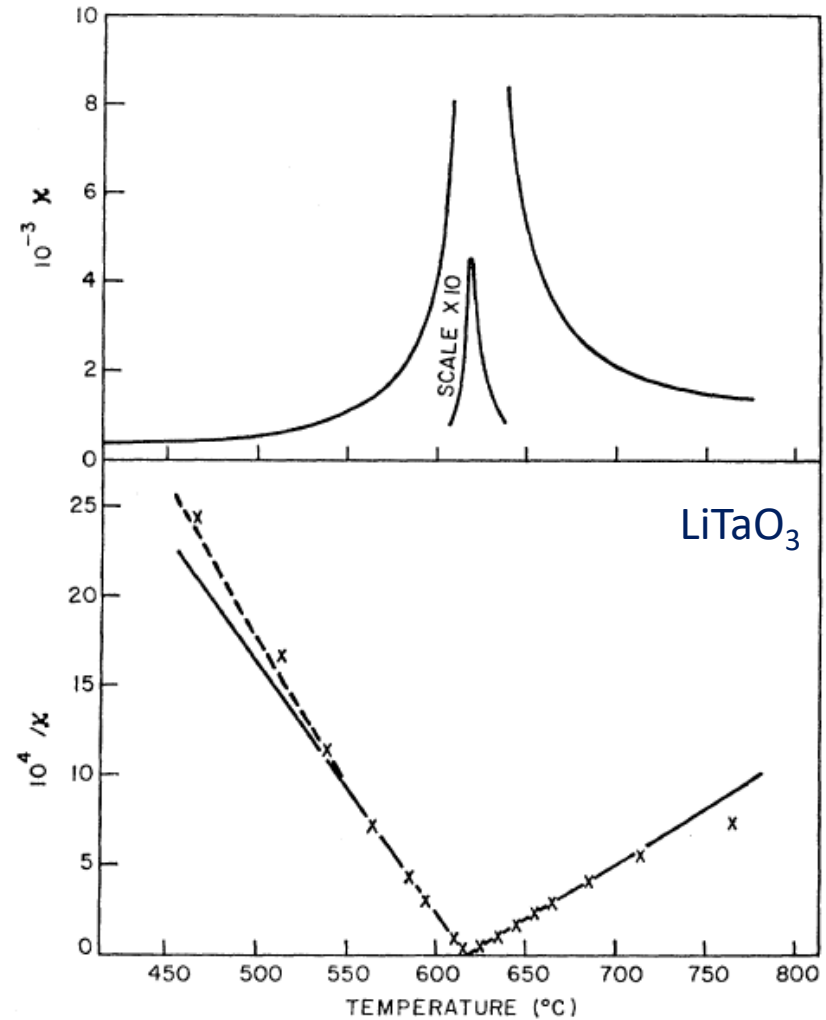
$$\chi\epsilon_0 = \frac{\partial P}{\partial E}$$

$$E = \alpha P + \beta P^3$$

$$(\chi\epsilon_0)^{-1} = \alpha + 3\beta P^2$$

$$= \begin{cases} \alpha & T > T_0 \\ -2\alpha & T < T_0 \end{cases}$$

$$\chi \propto \frac{1}{T - T_0} \quad \text{Curie-Weiss}$$



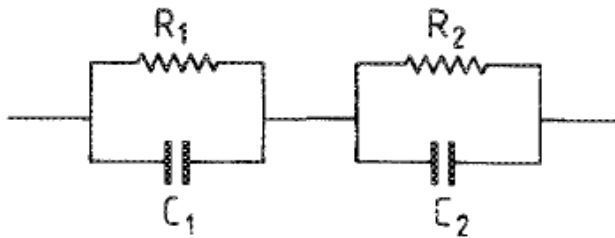
Glass, Phys. Rev. 1968

Dielectric constant diverges at 2<sup>nd</sup> order PT



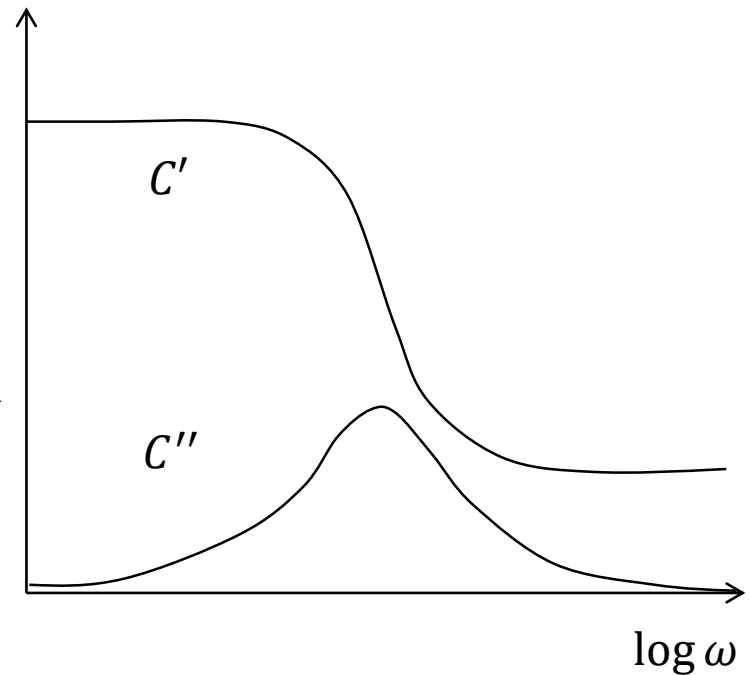
Often have a two (or more) component system, e.g. bulk + grain boundaries + metal/dielectric interface, Schottky barrier capacitance, etc...

Different  $C_i, R_i$  have different temperature dependences that combine to give a complex, frequency dependent dielectric response



$$Z(\omega) = Z' - iZ'' = \left( \frac{1}{R_1} + i\omega C_1 \right)^{-1} + \left( \frac{1}{R_2} + i\omega C_2 \right)^{-1}$$

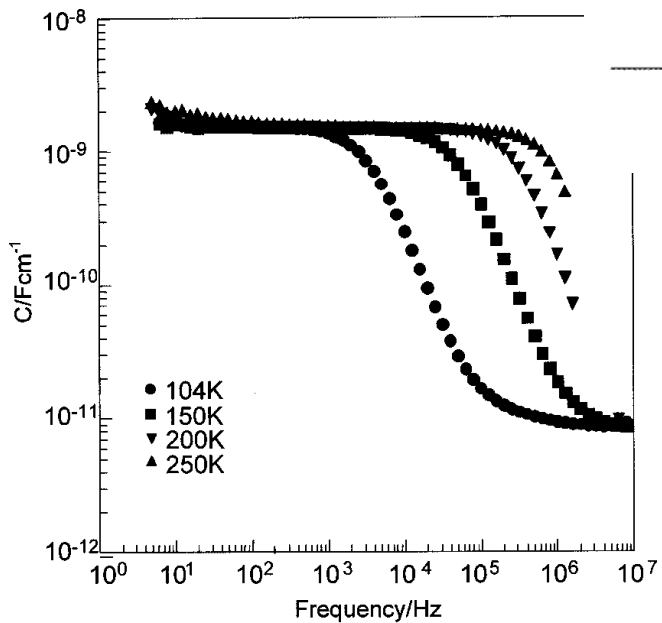
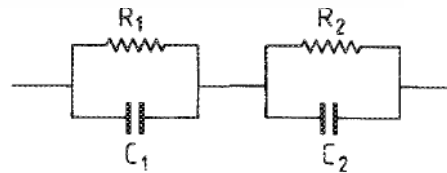
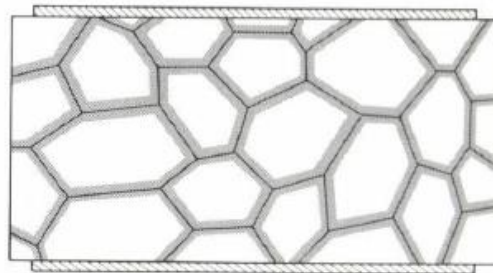
$$C(\omega) = C' - iC'' \equiv \frac{1}{i\omega Z}$$



Relaxations also due to space charge, domain wall motion etc.

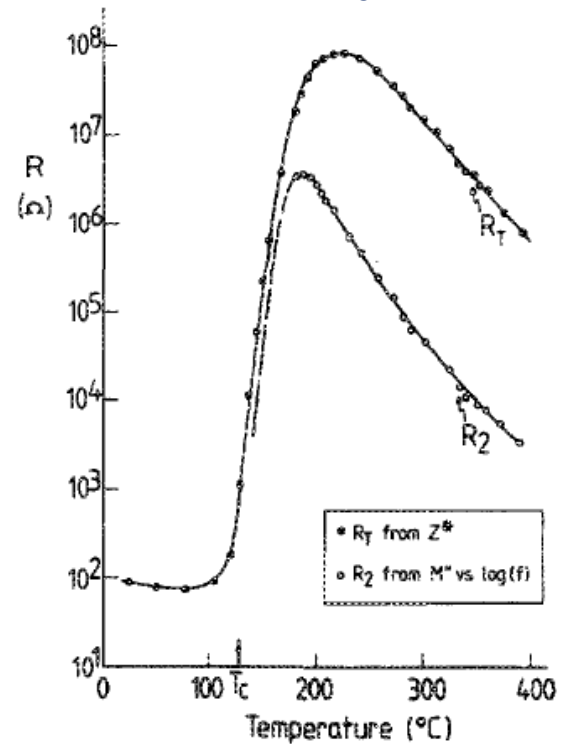
Always measure the frequency dependence!

Mechanism for Internal Barrier Layer Capacitors (IBLC)



'giant dielectric constant' of  $\text{CaCu}_3\text{Ti}_4\text{O}_{12}$   
Sinclair et al. APL 2002

Internal barrier structure important for PTCR  $\text{BaTiO}_3$  ceramics

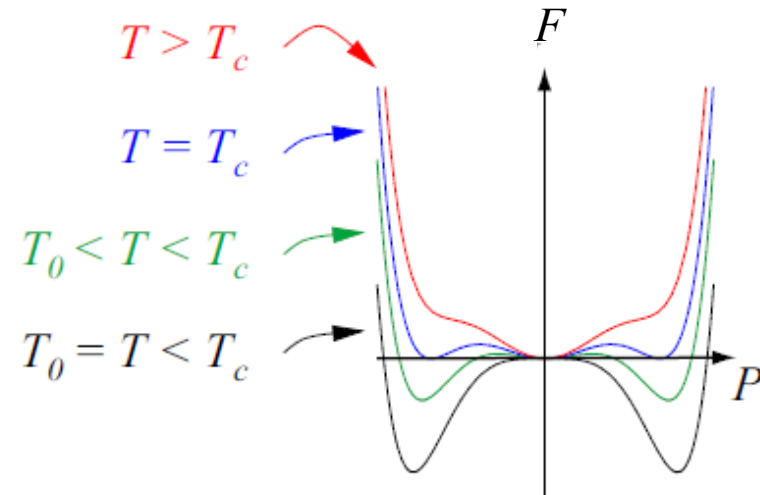


Sinclair & West JAP 1989

If  $\beta < 0$ , need to include  $P^6$  term.

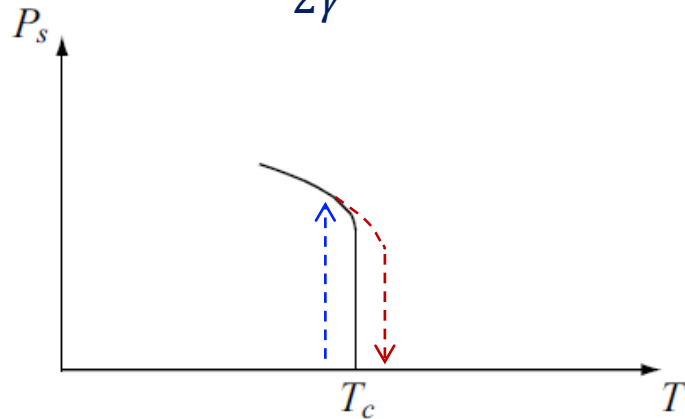
$$F = \frac{\alpha}{2}P^2 + \frac{\beta}{4}P^4 + \frac{\gamma}{6}P^6$$

$$T_C = T_0 + \frac{3}{16} \frac{\beta^2}{\alpha_0 \gamma} \quad T_C \neq T_0$$

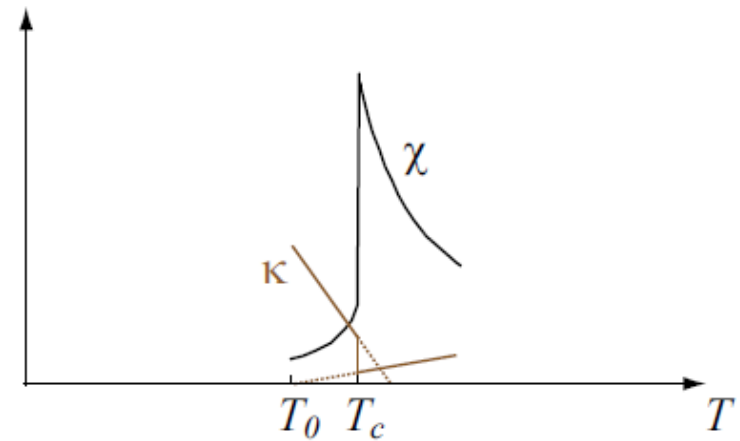


Images: N. Stucki

$$P_S^2 = \frac{-\beta + \sqrt{\beta^2 + 4\gamma\alpha_0(T - T_0)}}{2\gamma} \neq 0 \text{ at } T_C$$



Discontinuous order parameter  
→ thermal hysteresis



No divergence

Many ferroelectrics are weakly first order in bulk (e.g.  $\text{PbTiO}_3$ ,  $\text{BaTiO}_3$ )

Ferroelectric phase transitions often classified as being displacive or order-disorder:

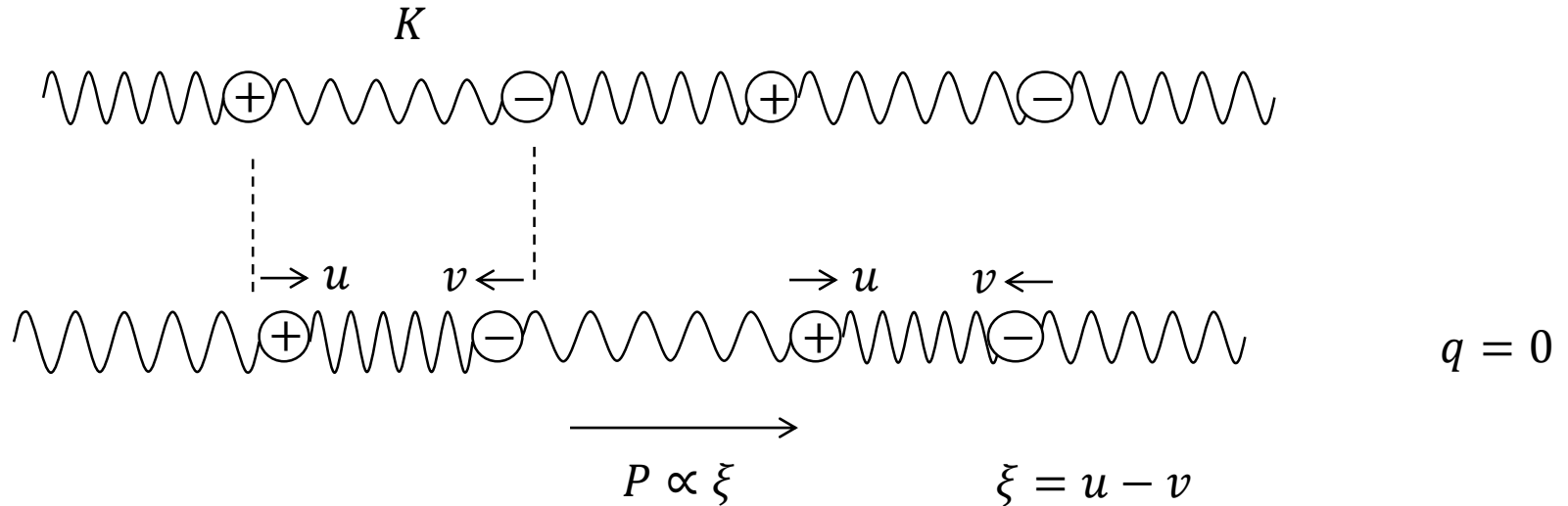
- Displacive – symmetry change results from displacement of atoms with respect to the more symmetric phase
- Order-disorder – symmetry change at transition results from redistribution of atoms over equiprobable positions

Generally, phase transitions have both displacive and order-disorder character



Cochran 1960

Toy model:



$q = 0$  ionic displacements

$\rightarrow P \propto \xi$

$\rightarrow$  local field  $E_{loc} \propto P \propto \xi$

$\rightarrow$  force =  $C\xi$

$$m\ddot{\xi} = -K\xi + C\xi$$

Short range

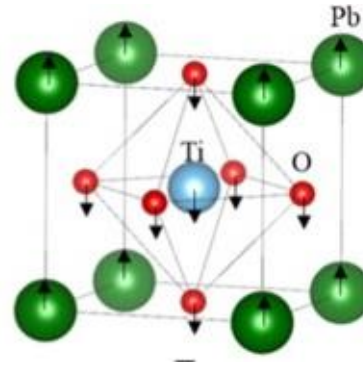
long range dipole-dipole interactions

$$\omega^2 = \frac{K - C}{m} \quad (\text{TO phonon})$$

## Displacive transition in $\text{PbTiO}_3$

$\omega^2 \propto (T - T_0)$  Mode softens!

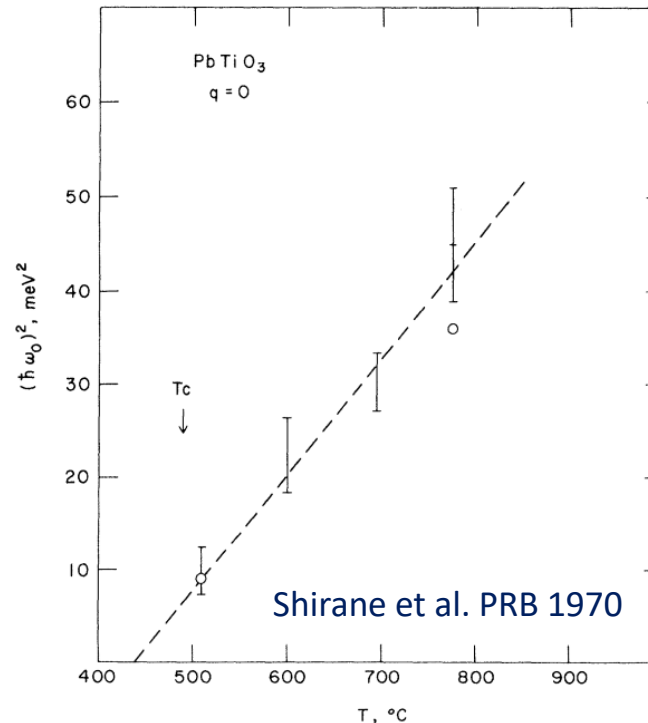
Below  $T_C$  freeze new equilibrium structure with finite  $\xi$



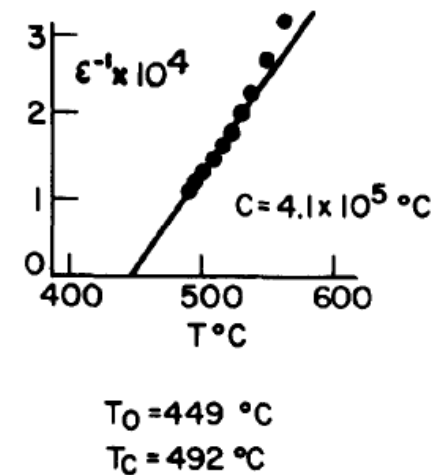
$$P = \frac{Z^*}{V} \xi_{FE}$$

Lyddanne-Sachs-Teller relation

$$\frac{\epsilon(0)}{\epsilon(\infty)} = \frac{\omega_L^2}{\omega_T^2} \propto \frac{1}{T - T_0}$$

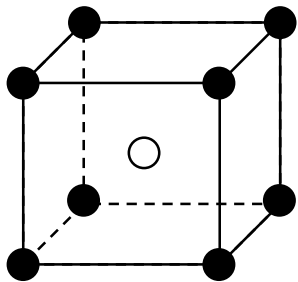


Remeika & Glass (1970)

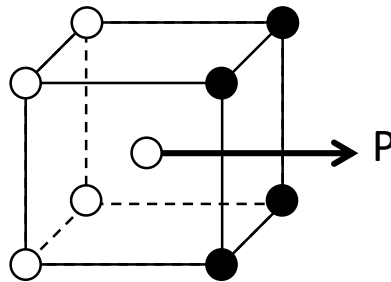


e.g. eight-site model for BaTiO<sub>3</sub>

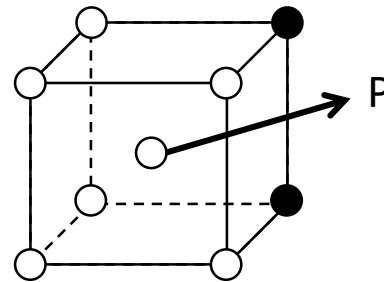
cubic



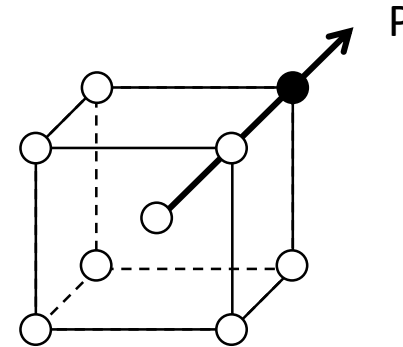
tetragonal



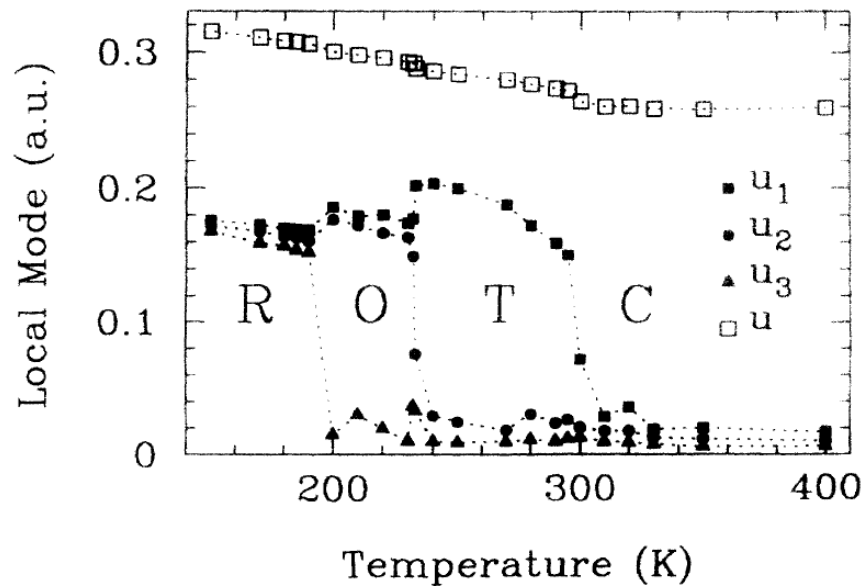
orthorhombic



rhombohedral



Comes Acta Cryst. 1970



Zhong, Vanderbilt, Rabe,  
PRL 1994

Generally, both displacive and order-disorder character present

$$F = \frac{\alpha}{2}P^2 + \frac{\beta}{4}P^4 + \dots + \frac{1}{2}C\eta^2 - g\eta P^2 - \eta\sigma - EP$$

In equilibrium  $\frac{\partial F}{\partial P} = 0, \frac{\partial F}{\partial \eta} = 0$

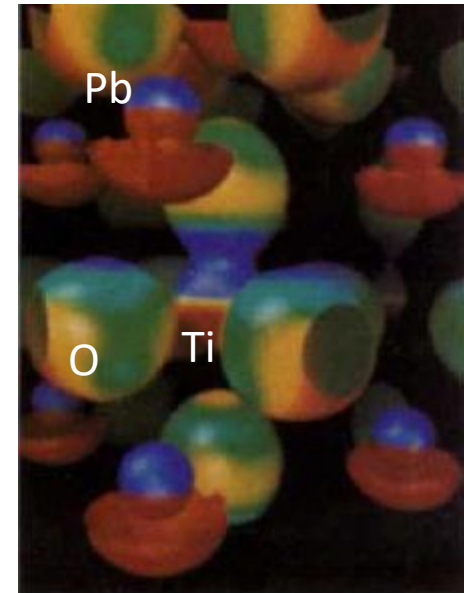
$$C\eta - gP^2 - \sigma = 0$$

Free crystal, fix  $\sigma = 0 \rightarrow \eta = \frac{gP^2}{c}$

### Spontaneous strain along $P$

Coupling to strain especially strong in  $\text{PbTiO}_3$   
due to polarisability of the lone pair on Pb

- $P \rightarrow P_i$  polarisation vector
- $C \rightarrow C_{ijkl}$  elastic compliance tensor
- $\eta \rightarrow \eta_{ij}$  strain tensor
- $g \rightarrow g_{ijkl}$  electrostrictive coefficients
- $\sigma \rightarrow \sigma_{ij}$  stress tensor



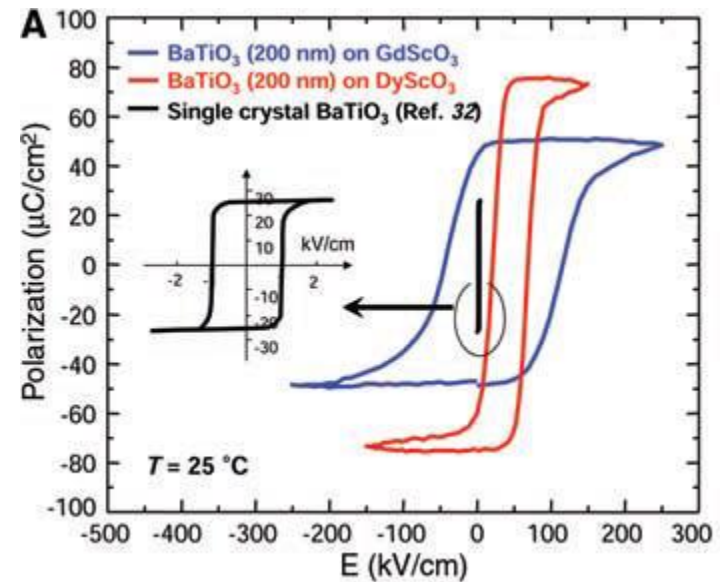
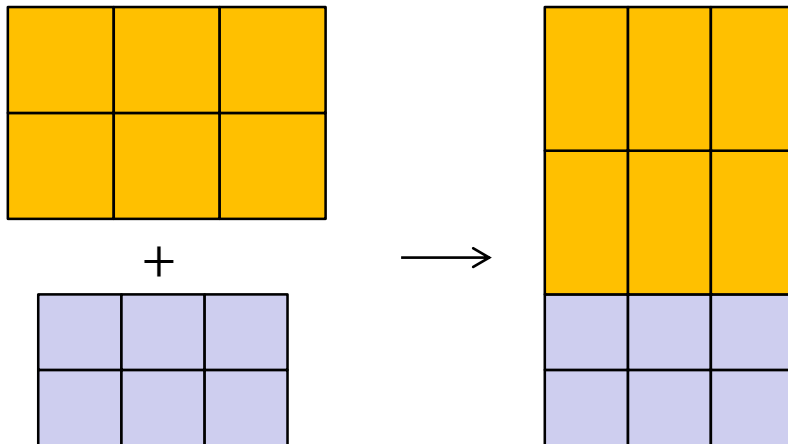
Cohen Nature 1992



$$F = \frac{\alpha}{2} P^2 + \frac{\beta}{4} P^4 + \dots + \frac{g^2}{2C} P^4 - \frac{g^2}{C} P^4 = \frac{\alpha}{2} P^2 + \left( \frac{\beta}{4} - \frac{g^2}{2C} \right) P^4 + \dots$$

Spontaneous strain renormalizes  $P^4$  coefficient! Can lead to change in the order of PT

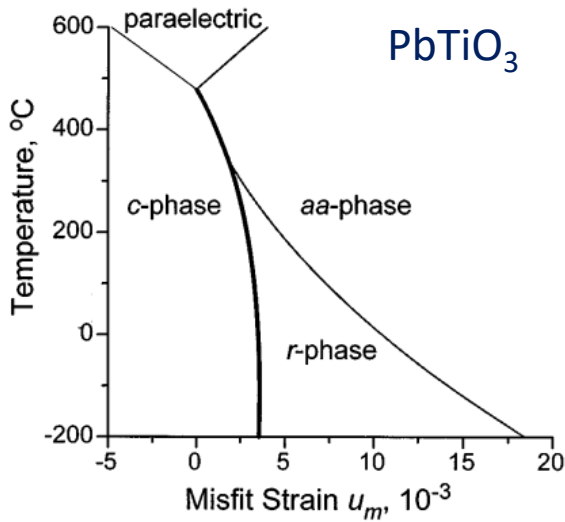
If instead we fix  $\eta$  (clamped crystal) – renormalize  $P^2$  coefficient – change  $T_C$ !



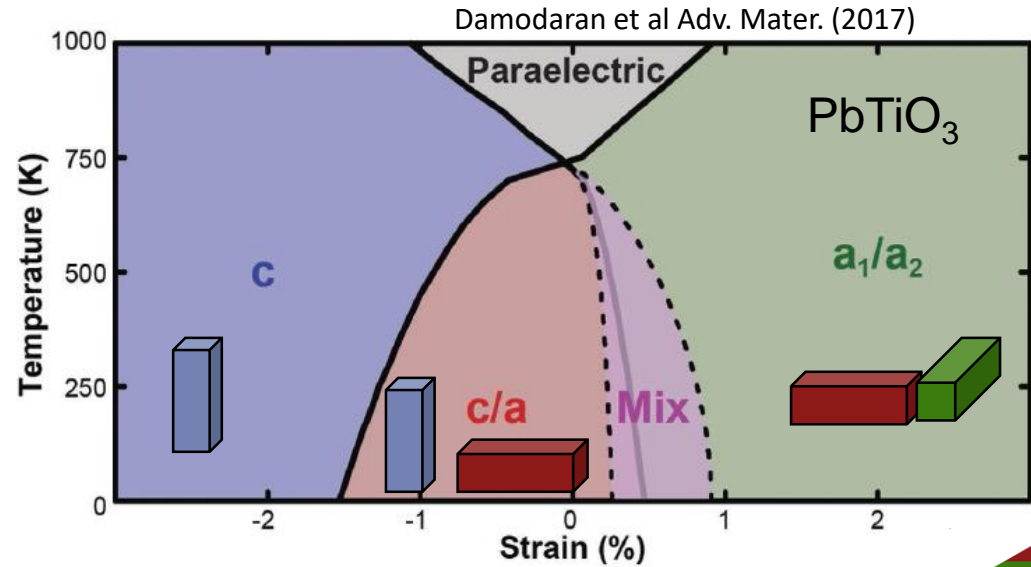
Choi et al. Science 2004

**BASIS FOR STRAIN ENGINEERING!**

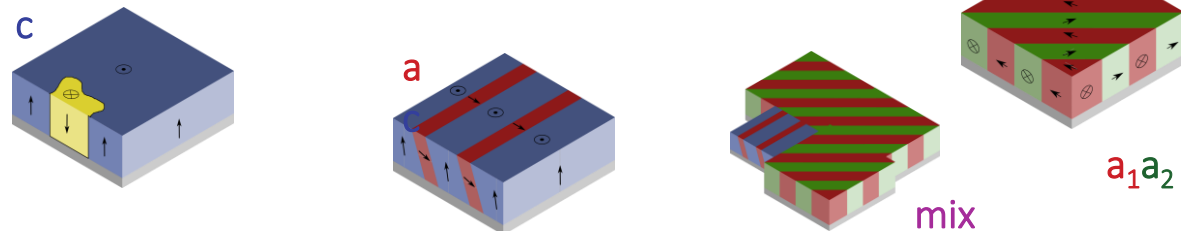
New phases unavailable in bulk!



Pertsev et al. PRL 1998



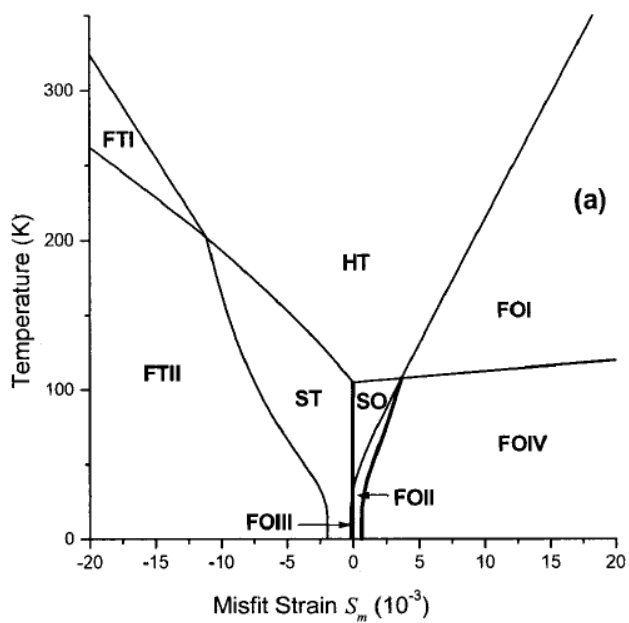
Damodaran et al Adv. Mater. (2017)



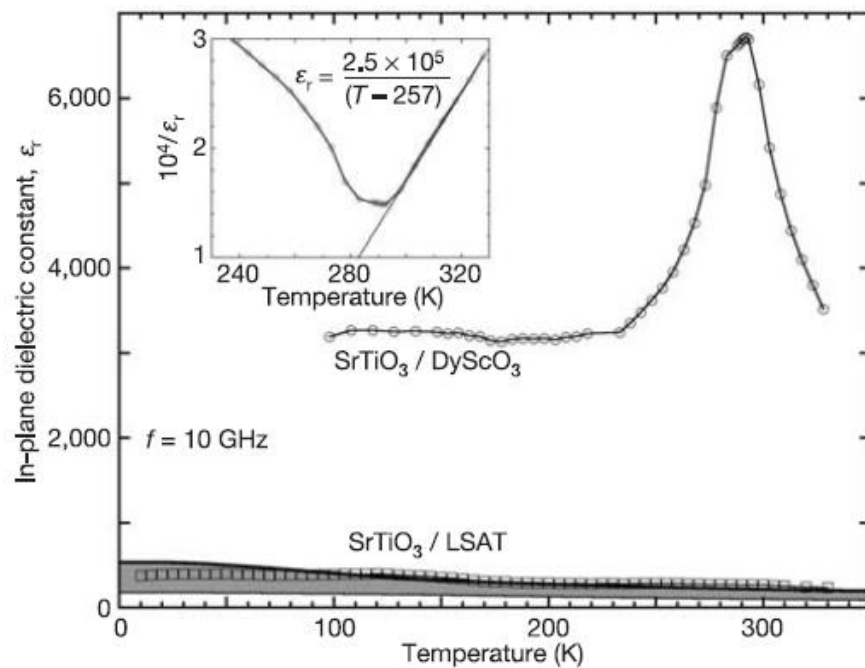
Koukhar et al. PRB 2001

See also review by Schlom et al. Annu. Rev. Mat. Res. 2007

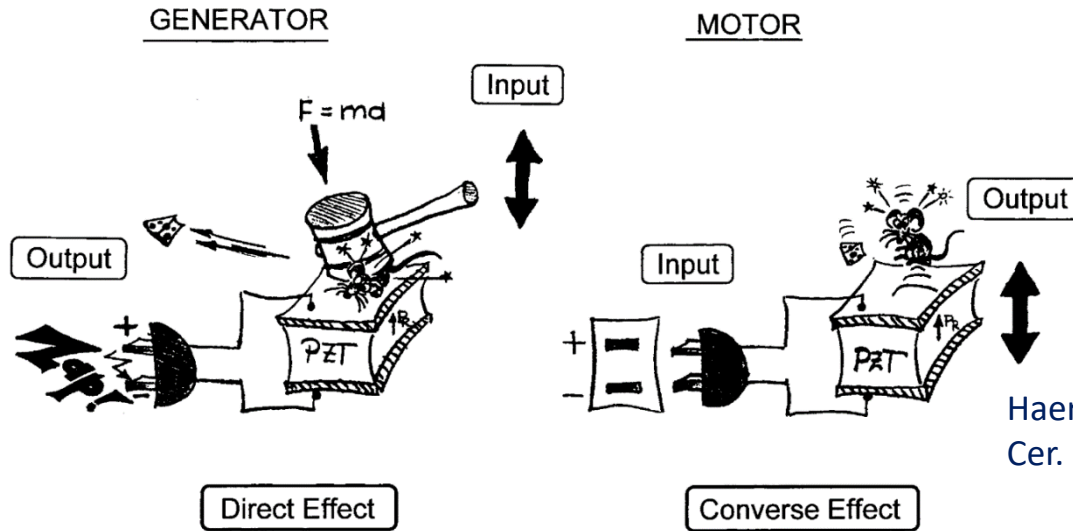
## Strain-induced ferroelectricity in SrTiO<sub>3</sub>



Pertsev et al. PRB 2000



Haeni et al. Nature 2004



Haertling J. Am. Cer. Soc. 1999

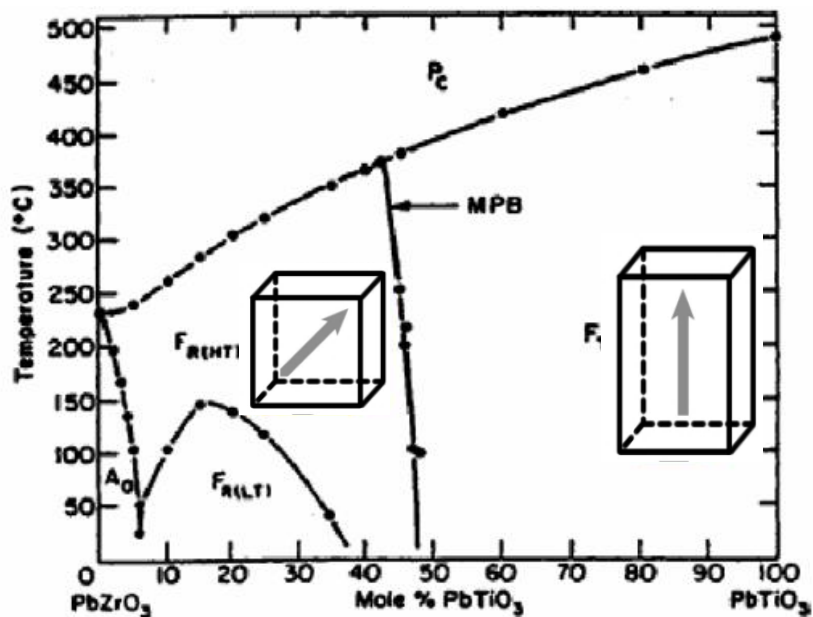
$$D = d\sigma$$

$$\eta = dE$$

Linear electromechanical coupling

$$d = \frac{\partial \eta}{\partial E} = \frac{\partial \eta}{\partial P} \frac{\partial P}{\partial E} = \left( -\frac{2gP}{C} \right) \chi \epsilon_0$$

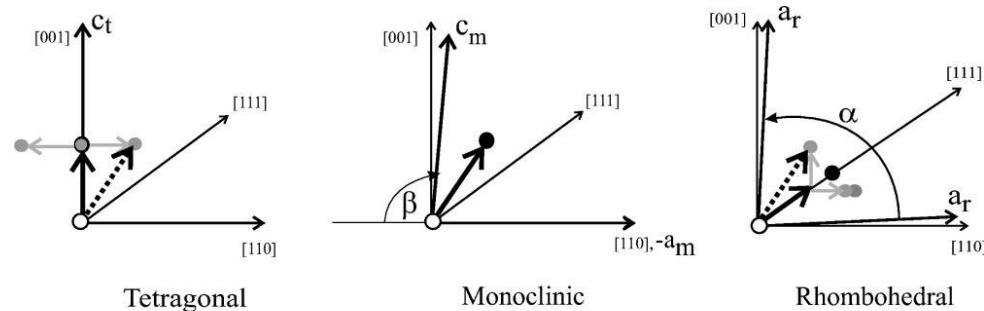
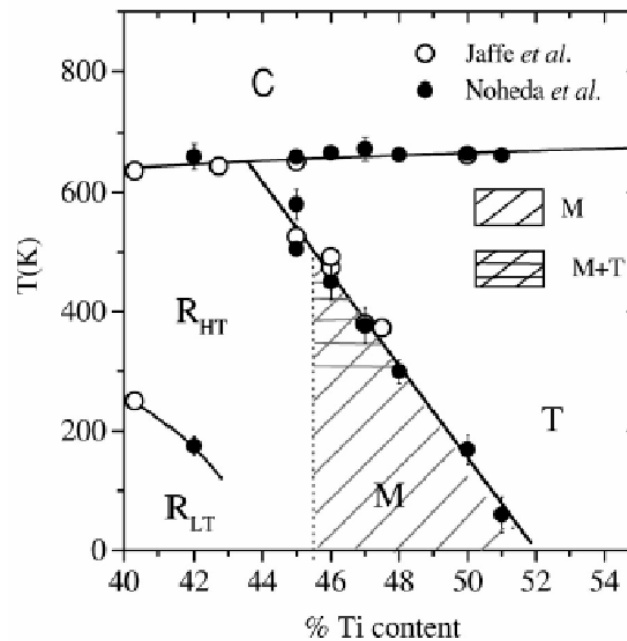




Jaffe 1971

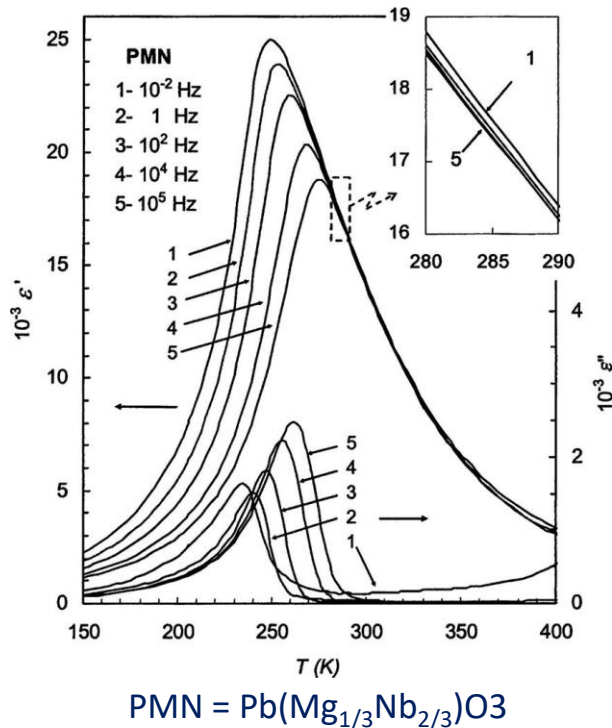
Dielectric and piezoelectric properties enhanced at morphotropic phase boundary (MPB)

Noheda PRB 2000



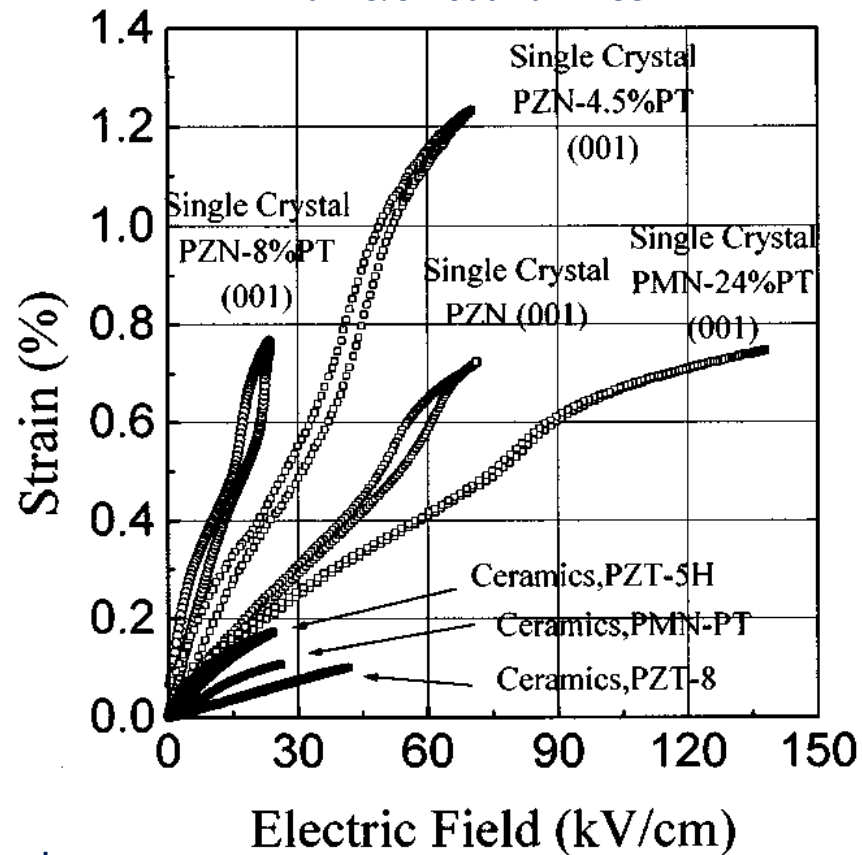
Intermediate monoclinic phase (M) facilitates polarisation rotation → large piezoresponse

Bokov, Ye – J. Mater. Sci. 2006



- Broad, frequency dependent dielectric anomaly
- No macroscopic structural phase transition
- Compositional disorder
- Mobile polar nanoregions (PNRs) that freeze below  $T_f$  or transition to ferroelectric state at  $T_C$

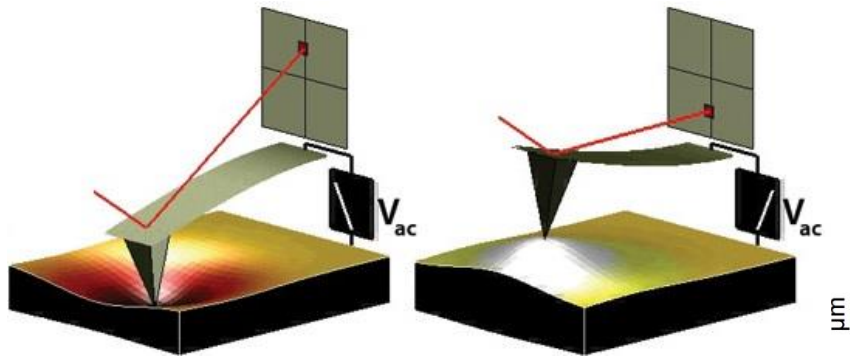
Park & Shrout – JAP 1997



- Giant electromechanical response!

See lecture by Brahim Dkhil

Exploit piezoresponse to probe ferroelectricity and ferroelectric domains using AFM

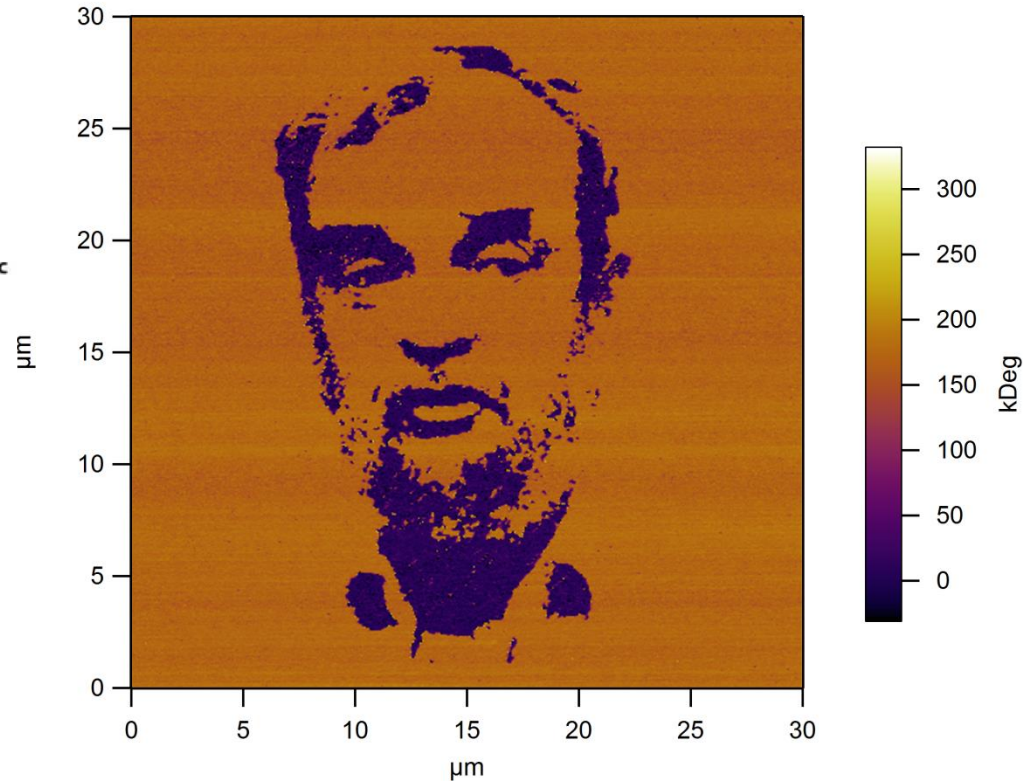


### Imaging:

- Apply voltage between tip and sample
- Sample expands or contracts piezoelectrically depending on local direction of  $P$

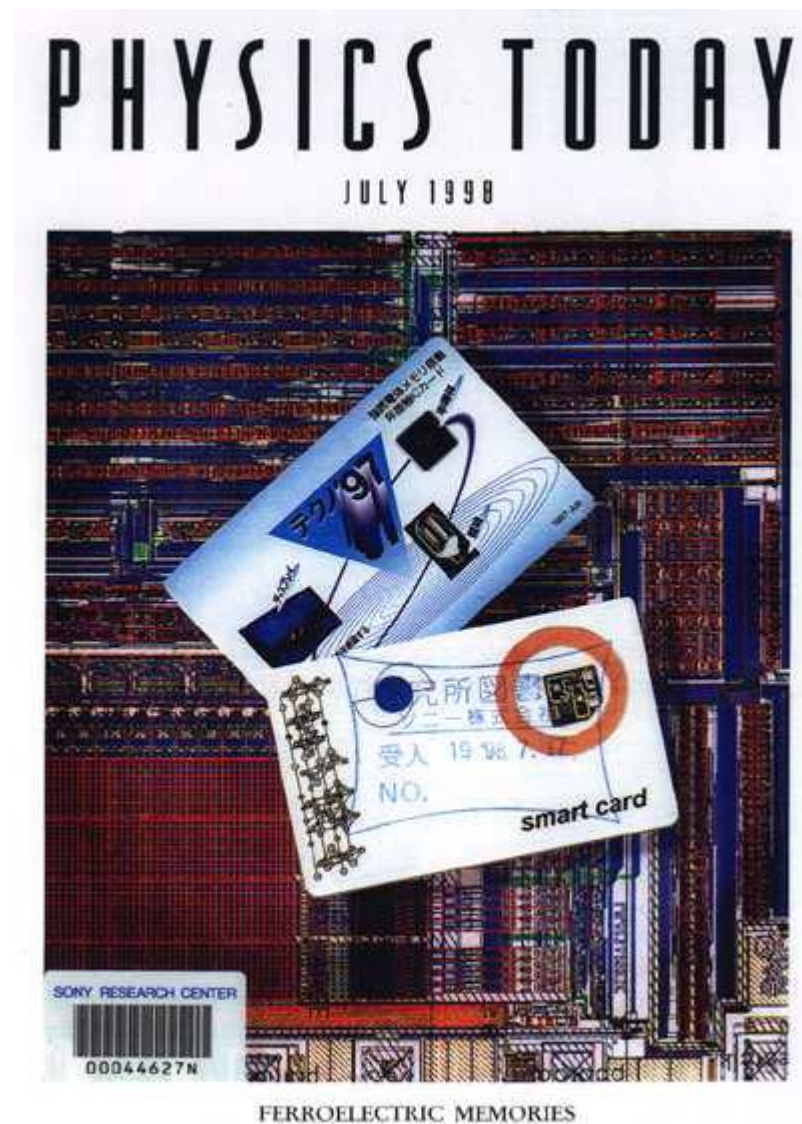
### Writing:

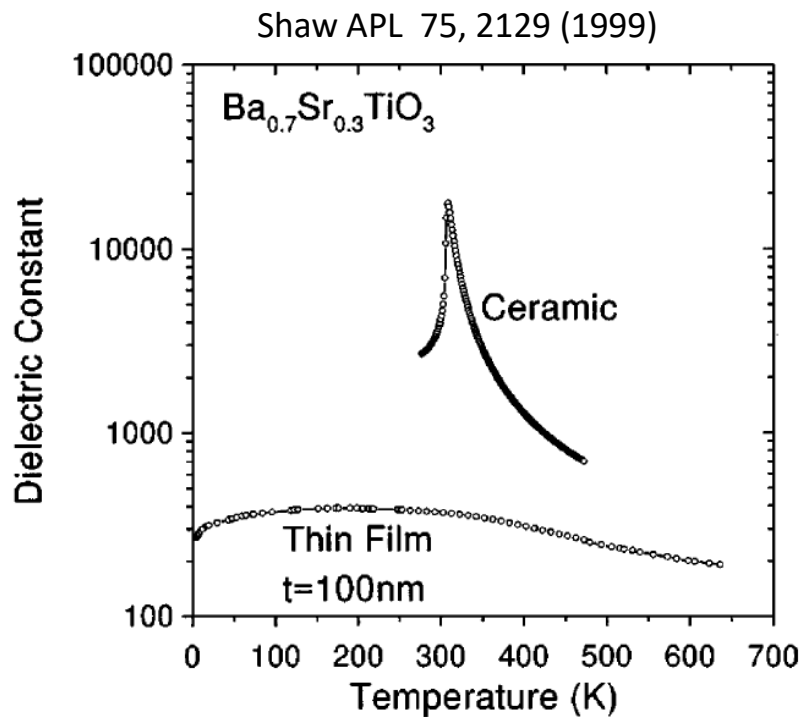
- Apply large enough voltage to switch  $P$



PFM microportrait of Jim Scott  
Canvas: BiFeO<sub>3</sub> thin film

- Finally enabled *ferroelectricity* to be exploited
  - FeRAM
  - Fe tunnel junctions
  - FeFET
- Must consider the properties of the whole system (M-F-M or M-F-S)
- Properties can be dominated by the interface rather than the ferroelectric itself



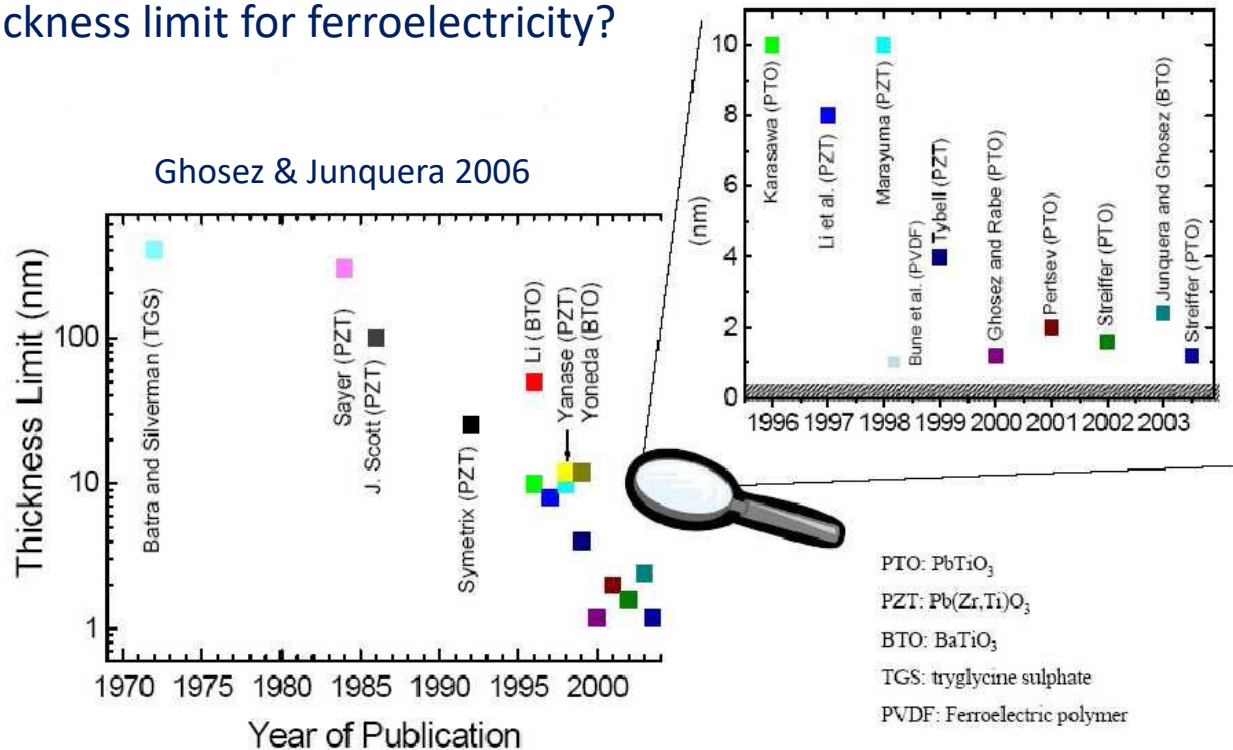


Thin films tend to have:

- smeared out transition
- reduced dielectric constants
- reduced/less stable polarisation



What is the thickness limit for ferroelectricity?



At surfaces

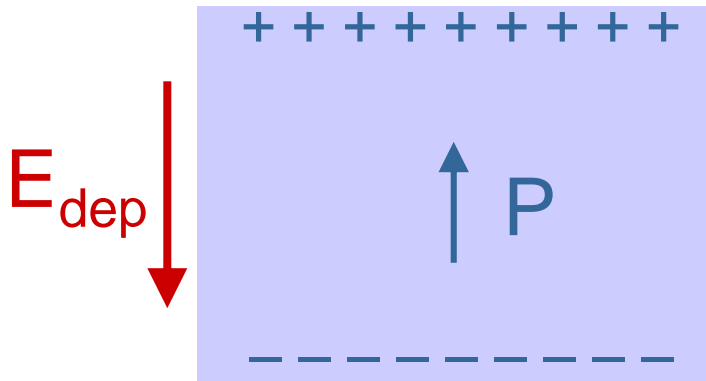
- Change in bonding
- Electrostatic boundary conditions
- Mechanical boundary conditions

Critical thickness for ferroelectricity has progressively decreased as better quality films have been made

Boundary conditions – very important!

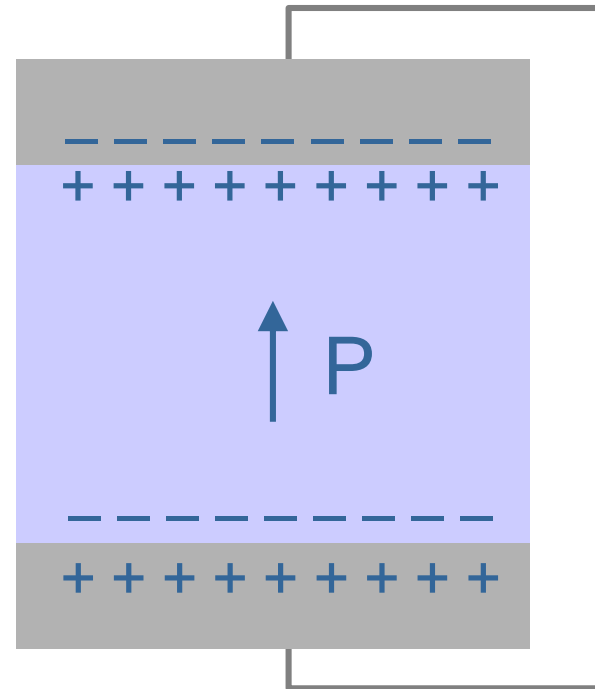
$$D = \epsilon_0 E + P$$

open circuit

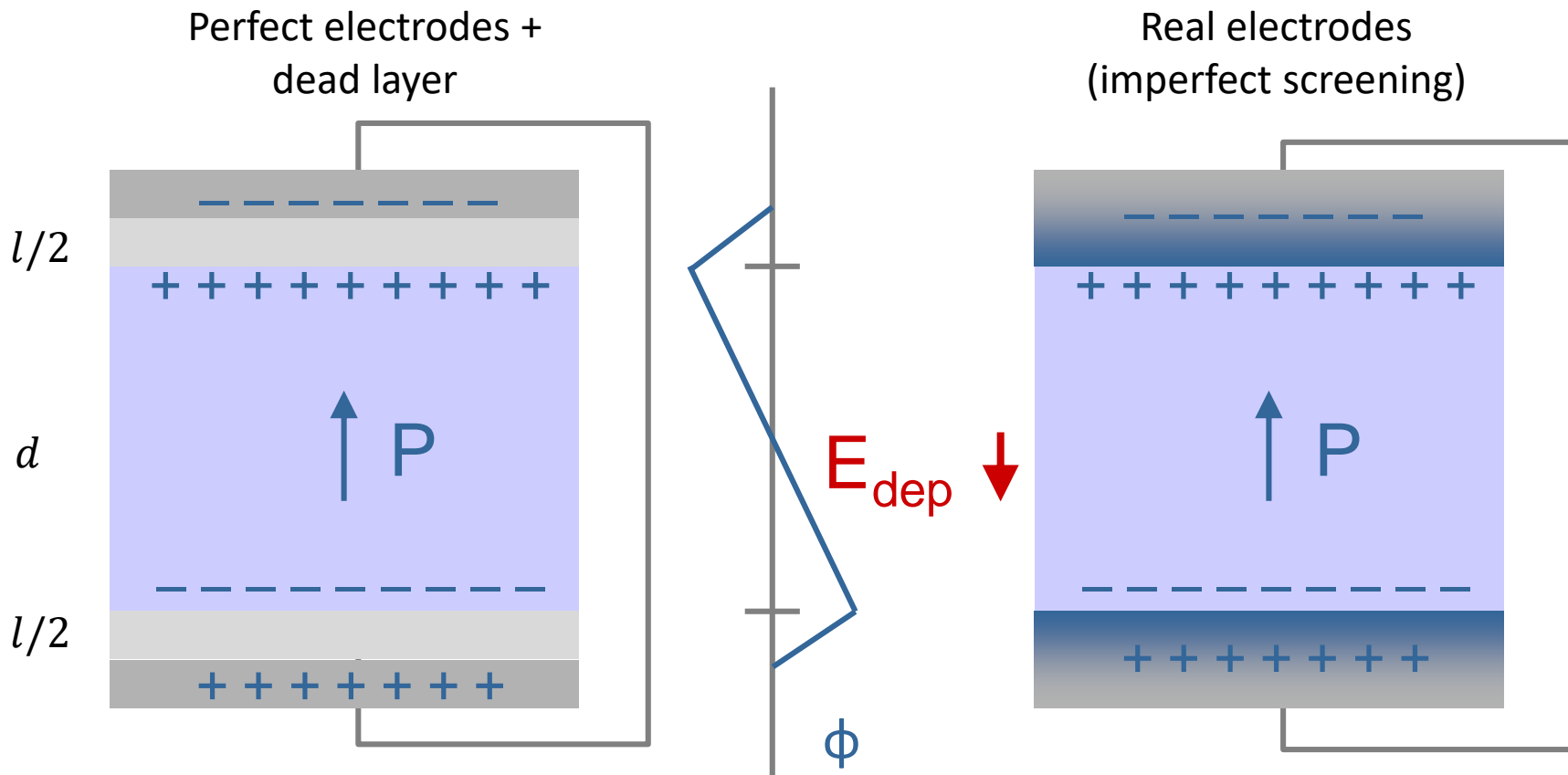


$$D = 0, E = -\frac{P}{\epsilon_0}$$

ideal short circuit



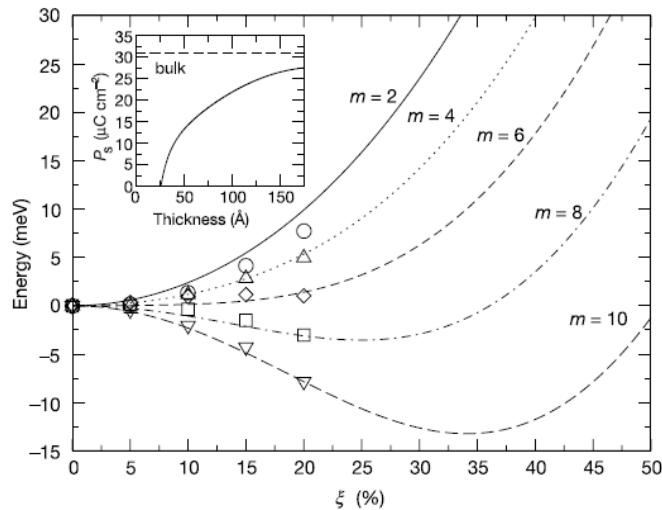
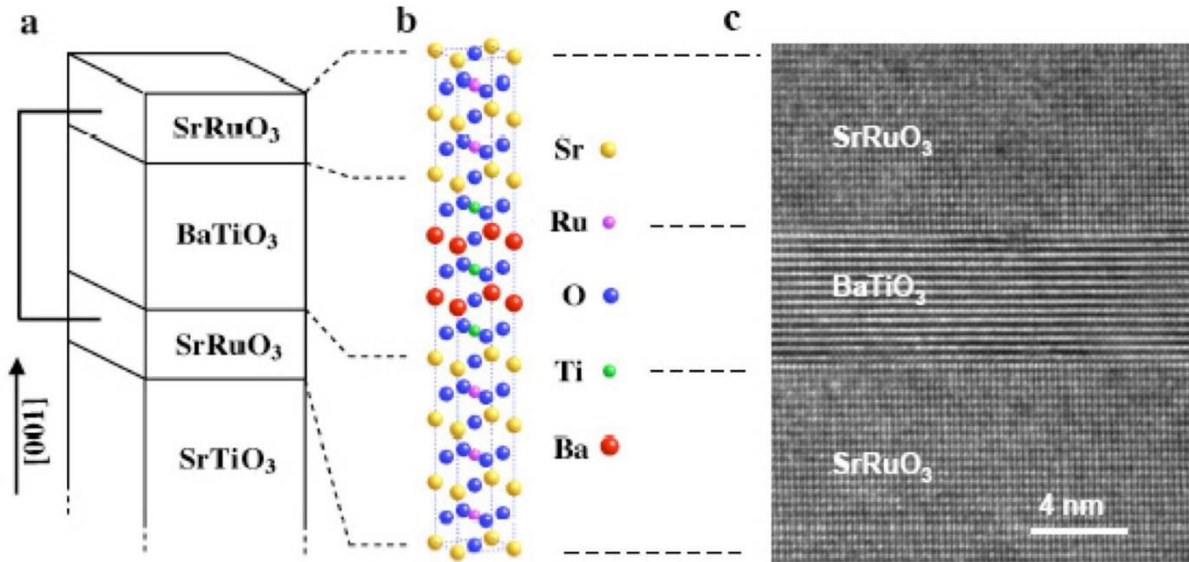
$$E = 0, D = P$$



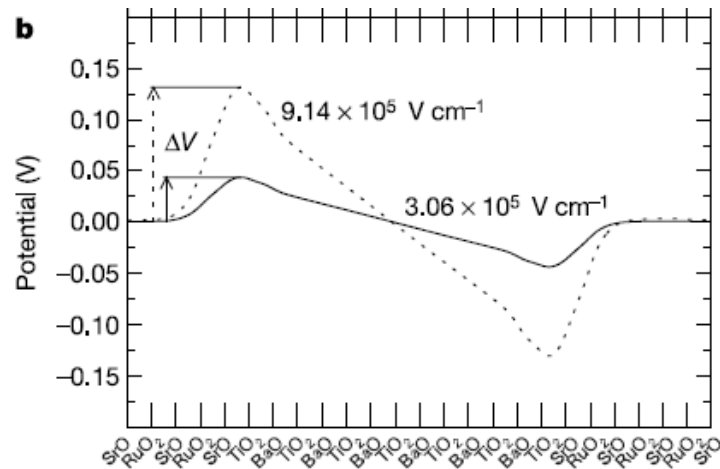
$$D = \epsilon_0 E_f + P_f = \epsilon_d \epsilon_0 E_d$$

$$\Delta\phi = E_d l + E_f d = 0$$

$$E_f = -\frac{l}{d} \frac{P_f}{\epsilon_d \epsilon_0} \equiv -\frac{\lambda_{eff}}{d} \frac{P_f}{\epsilon_0}$$

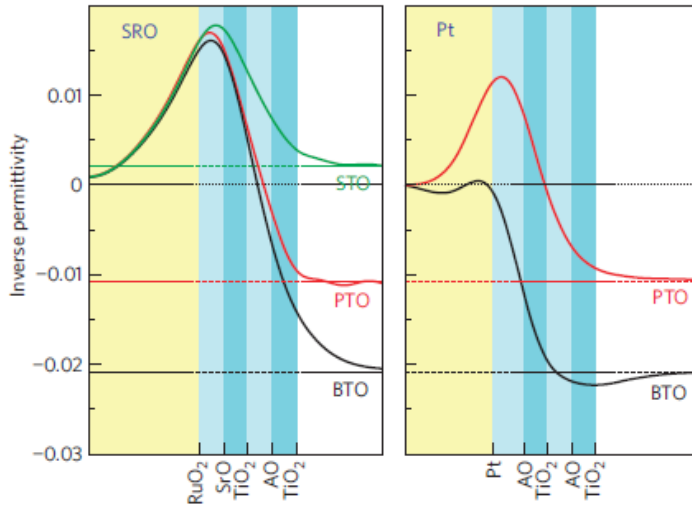


$$\lambda_{eff} \sim 10 - 20 \text{ pm}$$



Junquera, Ghosez Nature 2003

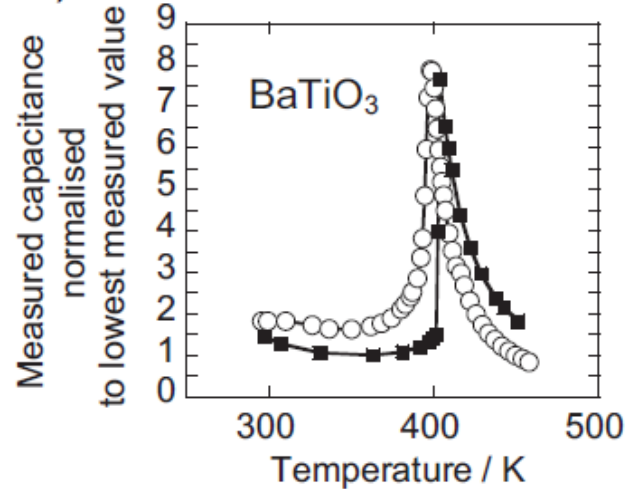
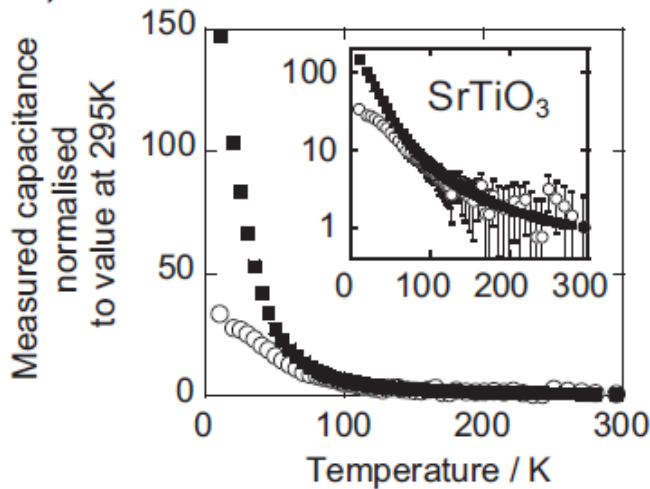
Theory



- For certain metal-ferroelectric combinations, 'dead layers' can be avoided
- Ferroelectricity can even be enhanced!

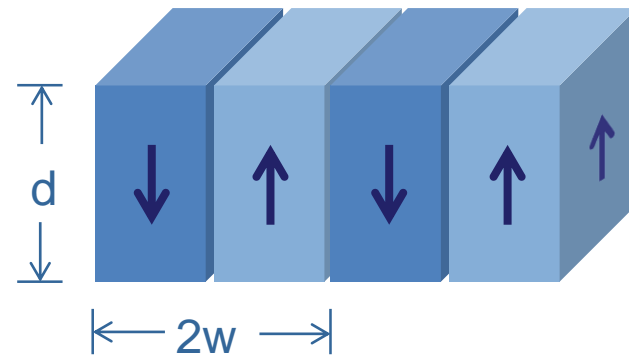
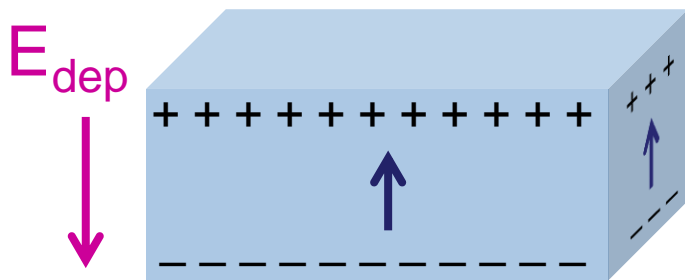
Stengel et al. Nature Mater. 2009

Experiment



Thin lamellae  
FIB-cut from  
single crystal  
○-○  
vs  
single crystal  
■-■

Chang et al. Adv. Mater. 2009

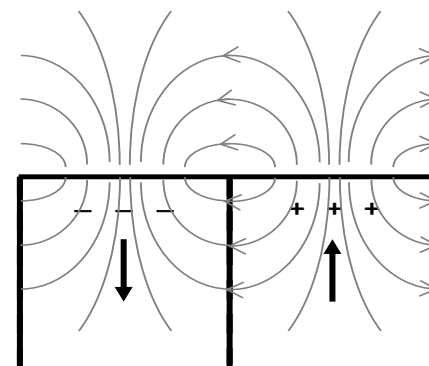


Domain wall energy ( $\sigma_{dw}$ ) vs. electric field energy ( $U$ )

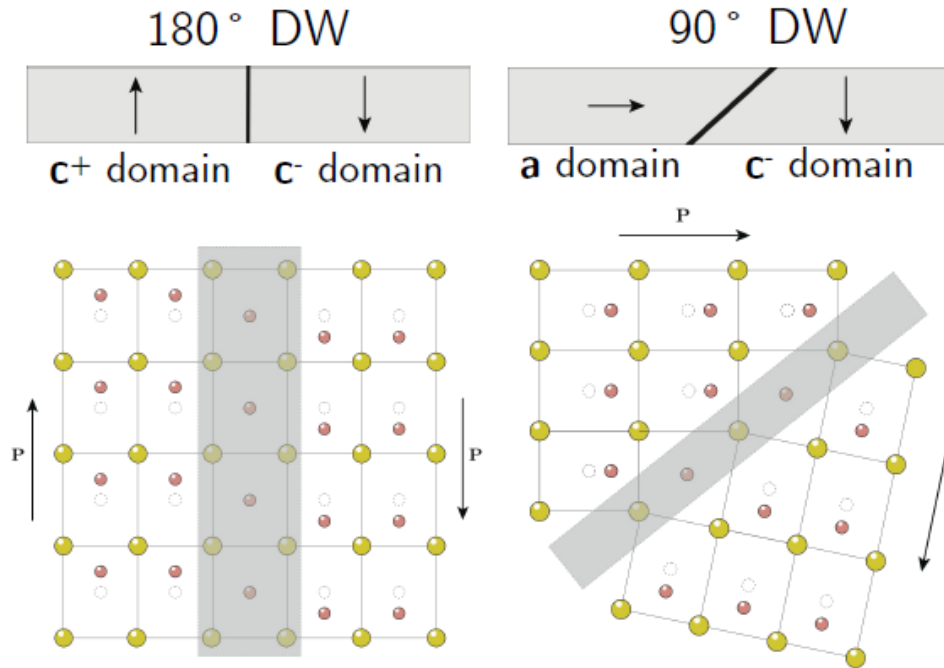
$$\Delta F = \sigma_{dw} \frac{d}{w} + Uw$$

$$w = \sqrt{\frac{\sigma_{dw}}{U}} d$$

Landau-Lifshitz-Kittel law







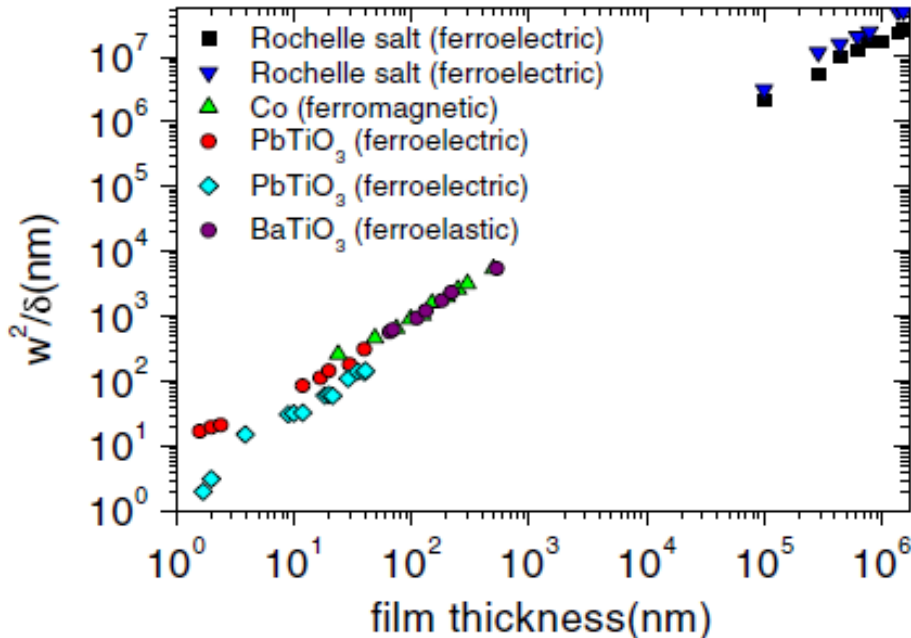
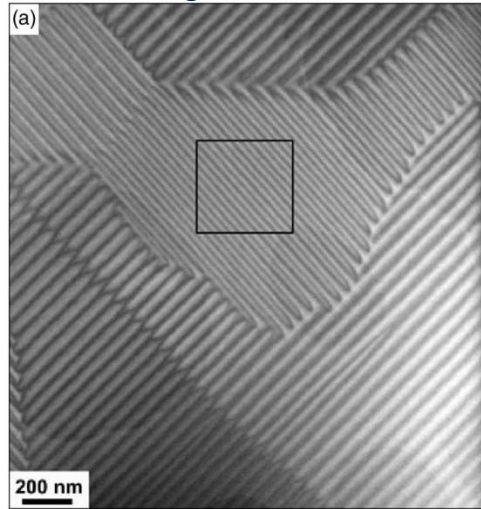
Respond to applied electric fields

Ferroelastic – respond to electric fields and mechanical stress

(The  $\sqrt{d}$  law also applies to ferroelastic domains)

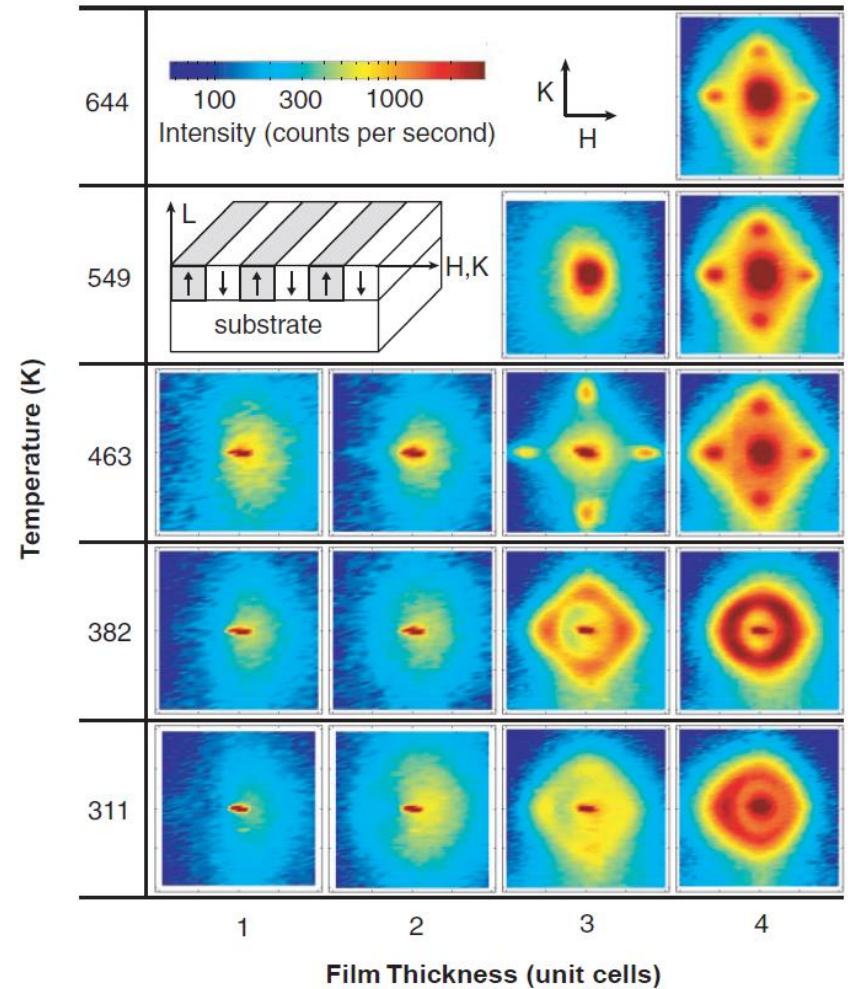
Schilling et al. PRB 2006

BaTiO<sub>3</sub>  
lamella  
(90°  
domains)



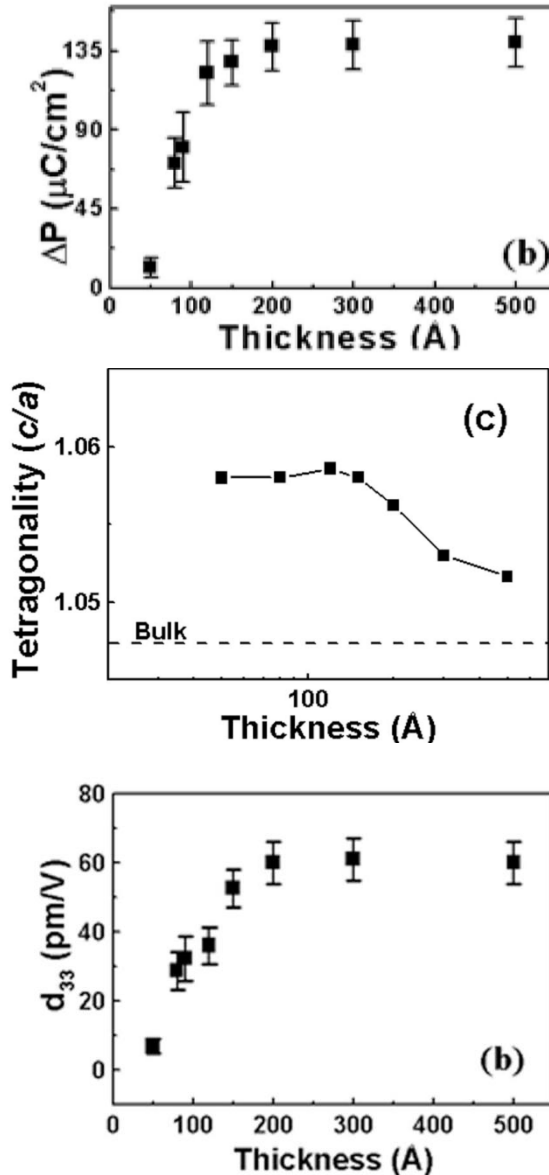
Catalan et al. Rev. Mod. Phys. 2012

Ultrathin PbTiO<sub>3</sub> films (180° domains)

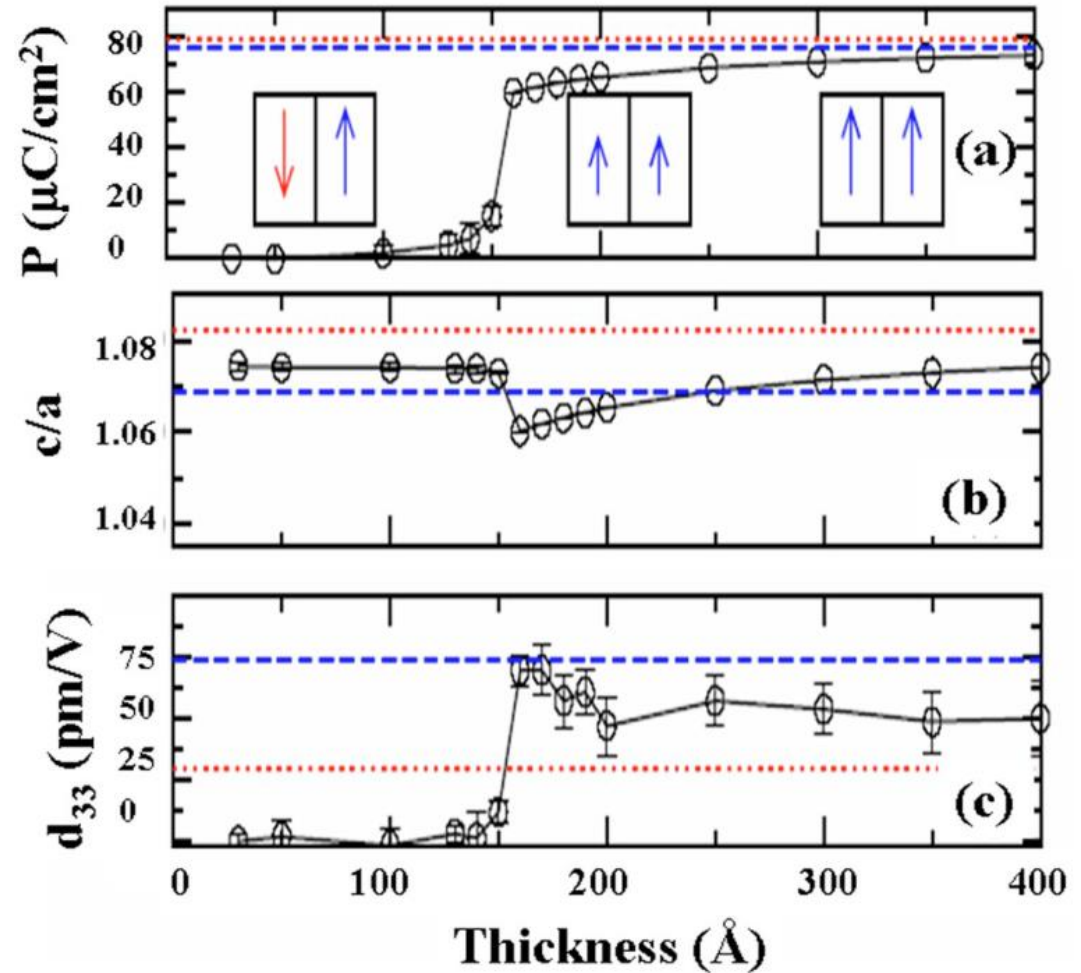


Fong et al. Science 2004

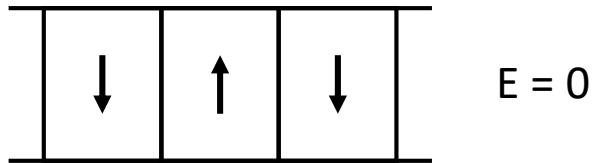
## Experiment (PZT capacitors)



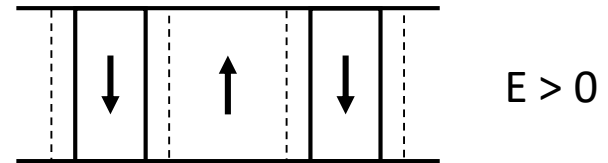
## Theory



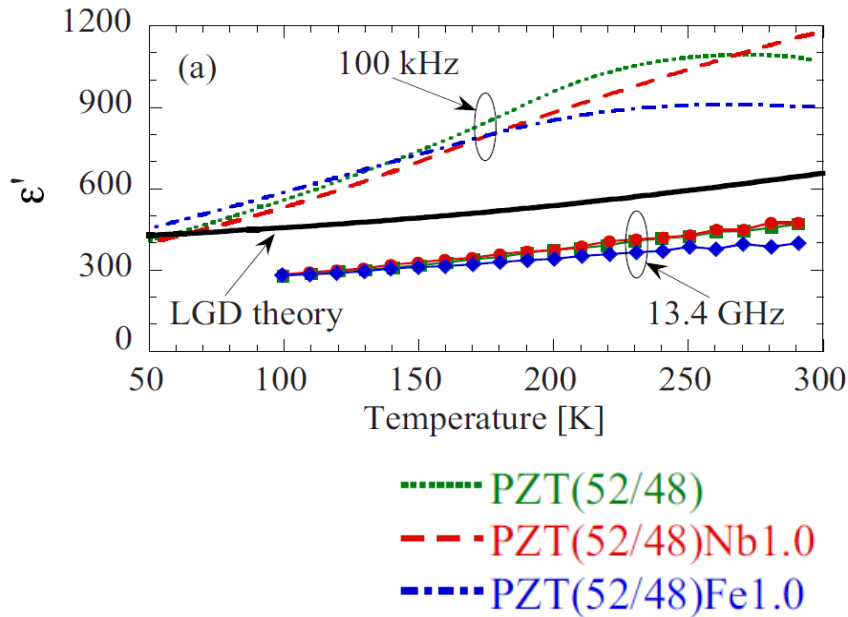
Nagarajan et al. JAP 2006



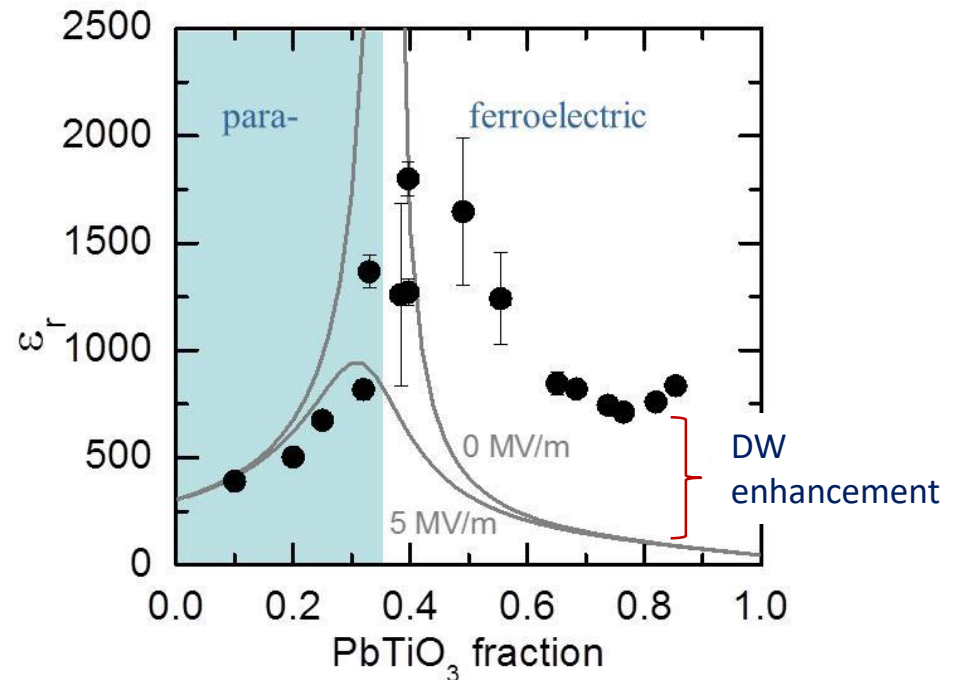
PZT ceramics



PbTiO<sub>3</sub>-SrTiO<sub>3</sub> superlattices



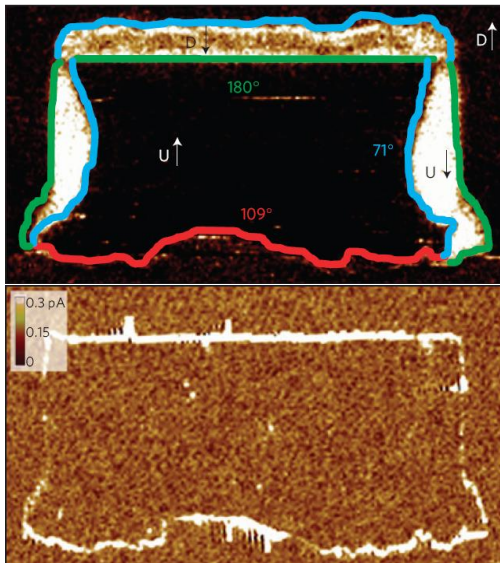
Lin, Damjanovic – APL 2010



PRL 104, 187601 (2010)

See also lecture on negative capacitance on Friday

Domain walls break symmetry of the bulk → new properties!



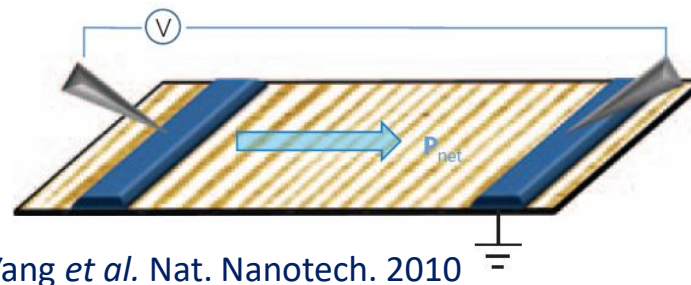
Seidel *et al.* Nat. Mater. 2009

Ferrielectric domain walls in  $\text{CaTiO}_3$

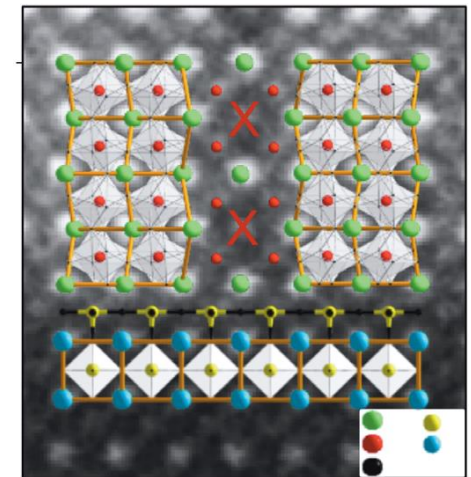
Polar domain walls in  $\text{SrTiO}_3$

Conducting domain walls in ferroelectrics

Photovoltaic response at domain walls in  $\text{BiFeO}_3$



Yang *et al.* Nat. Nanotech. 2010



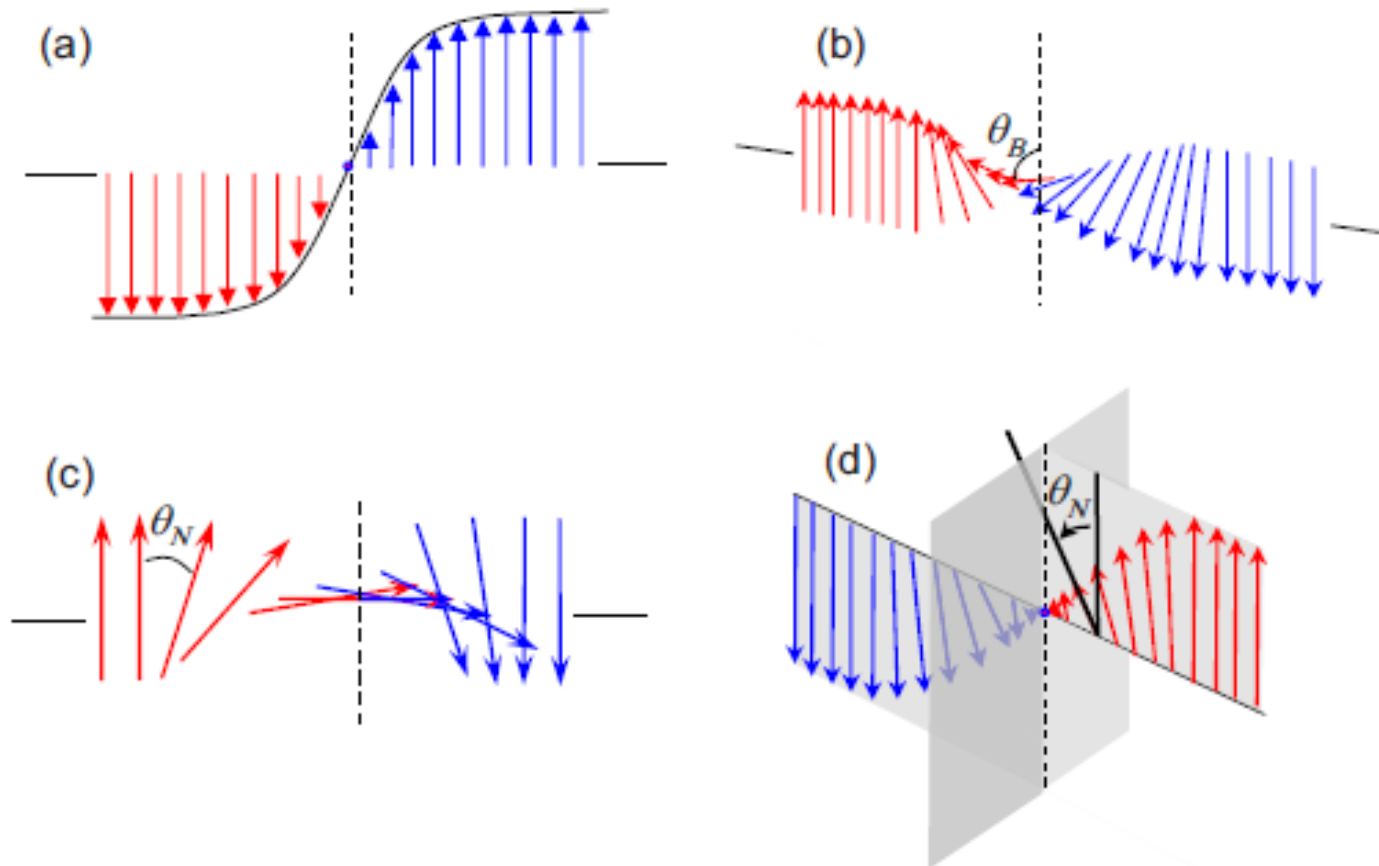
Farokhipoor *et al.* Nature 2015

Superconducting DWs in  $\text{WO}_{3-x}$

Magnetism at domain walls

“Domain Wall Nanoelectronics” – Catalan *et al.* Rev. Mod. Phys. 2012



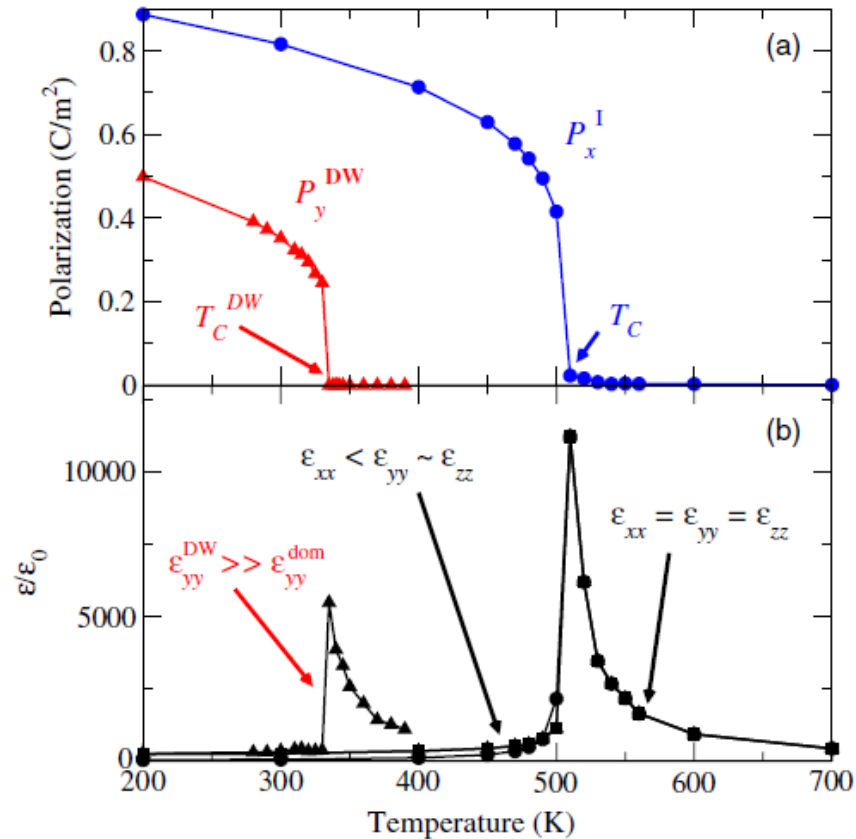
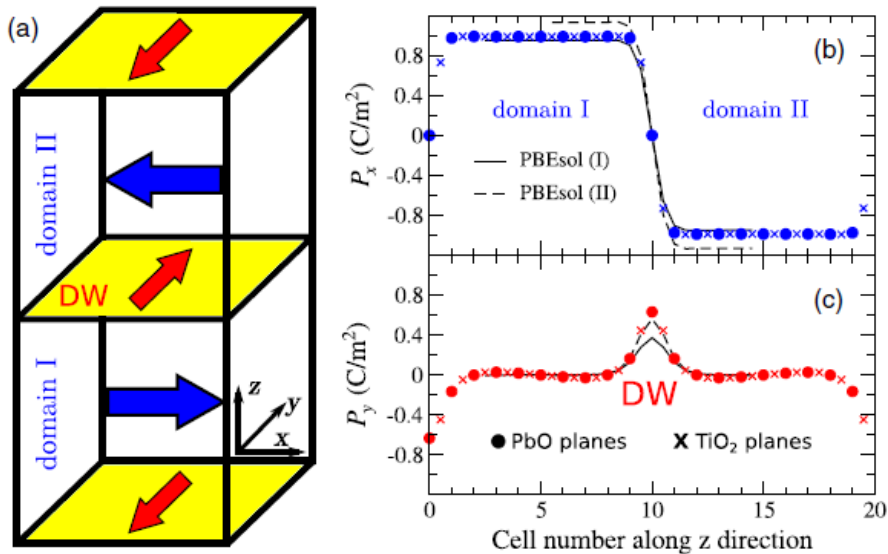


DW structure often not *purely* Ising-like

Lee et al. Phys. Rev. B 2009

(also Stepkova et al. JPCM 2012)

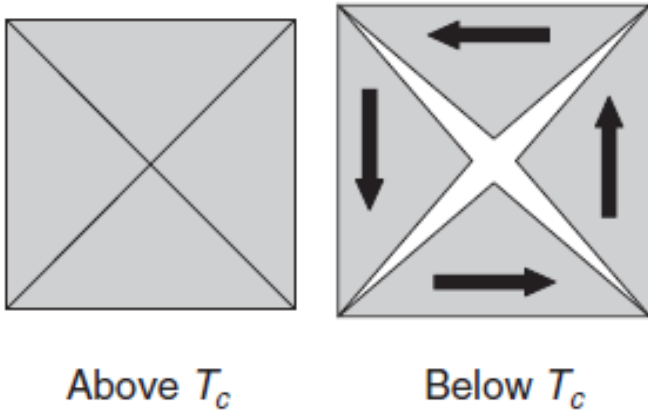




Many interesting theoretical predictions for domain walls and domain structures still to be experimentally explored!

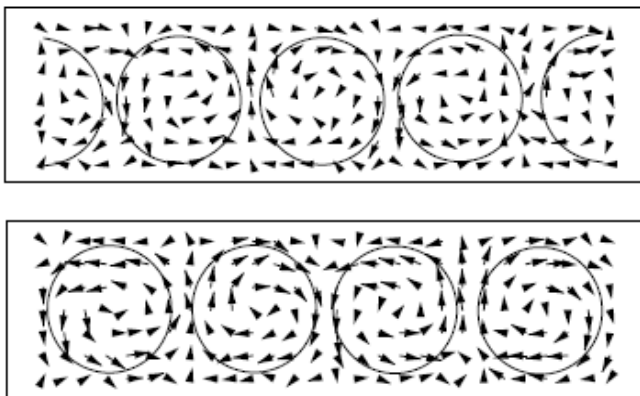
Wojdel & Iniguez, PRL 2014

Catalan et al. Rev Mod Phys 2011

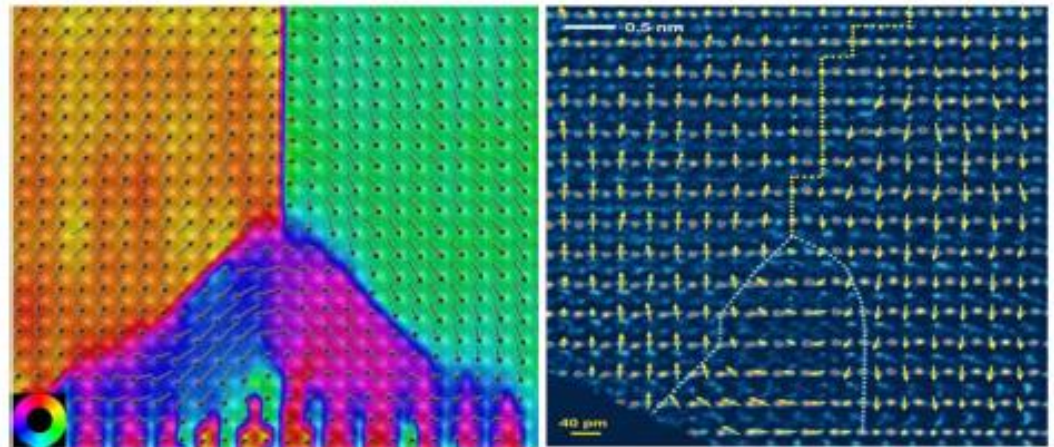


Large polarisation-strain coupling – large energy penalty for polarisation rotation in bulk

BUT in very thin films & nanostructures can have large local distortions - unusual polarisation structures possible



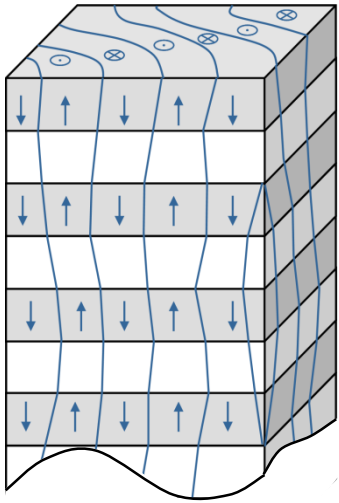
Naumov & Bellaiche Nature 2004



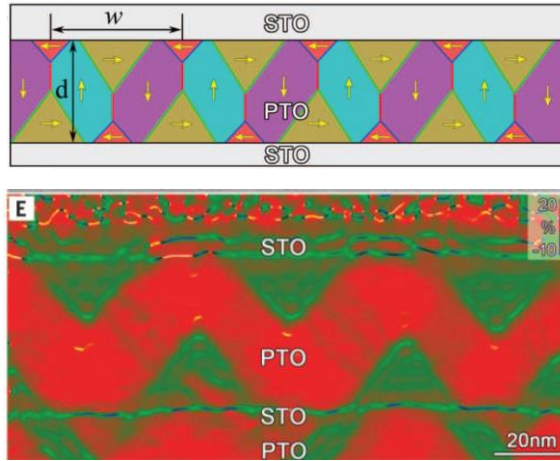
Nelson et al. Nano Letters 2011

Jia et al. Science 2011

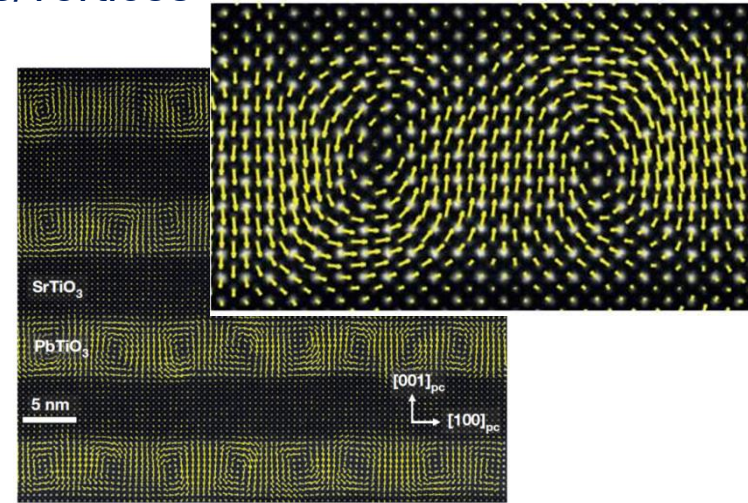
## 'Flux-closure domains/vortices'



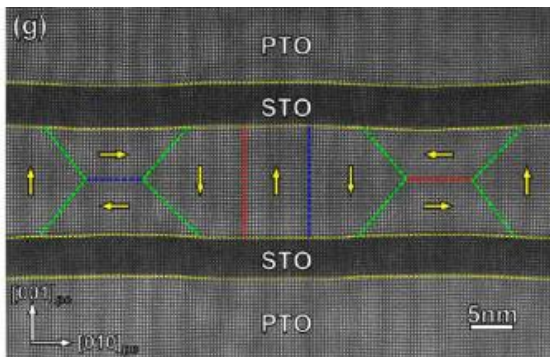
Zubko et al. PRL 2010;  
Nano Letters 2012



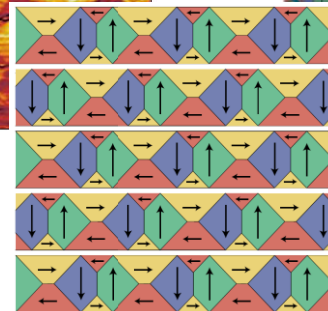
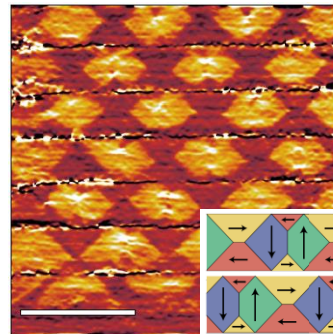
Tang *et al.*, Science. 2015



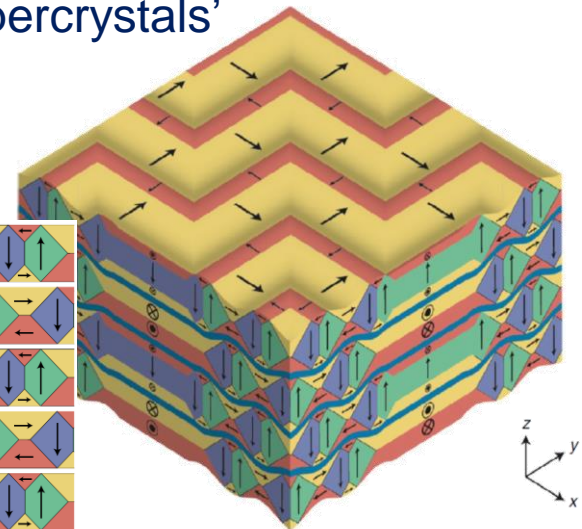
Yadav et al. Nature 2016



Liu et al. Nano Letters 17, 7258 (2017)

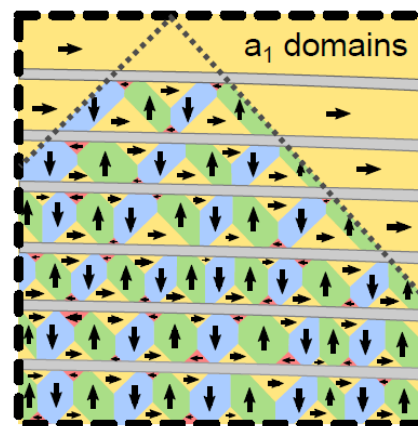
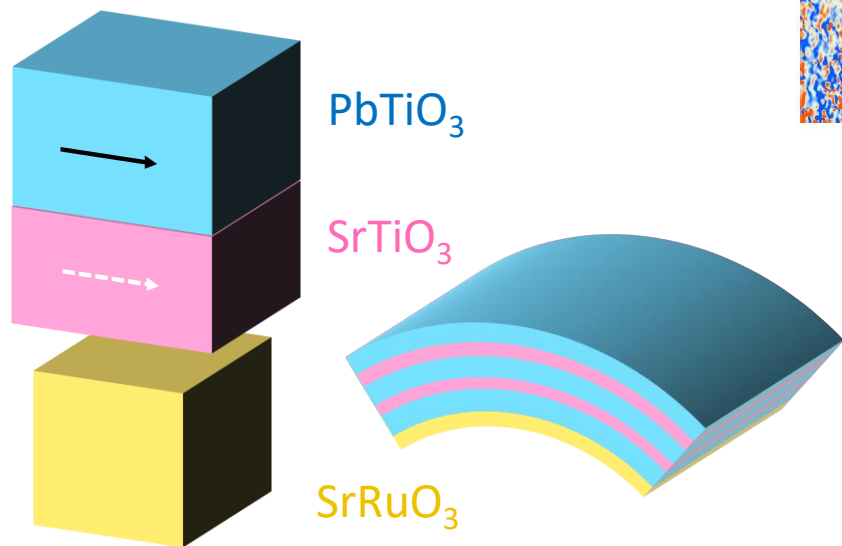
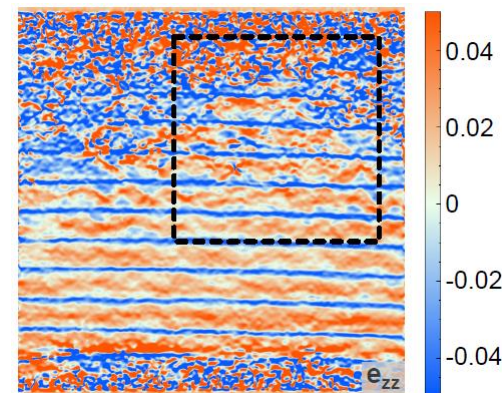
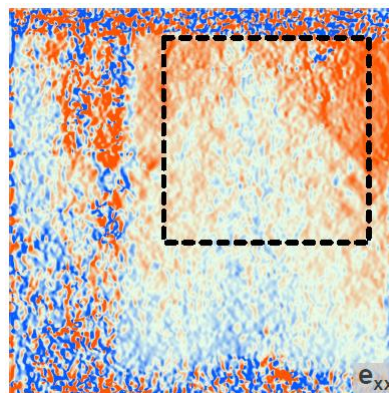
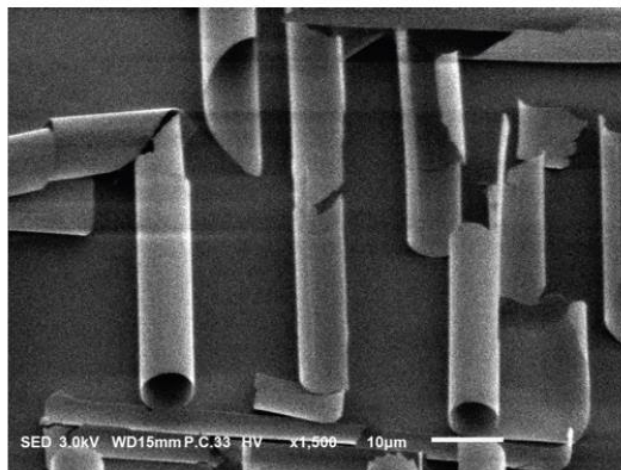


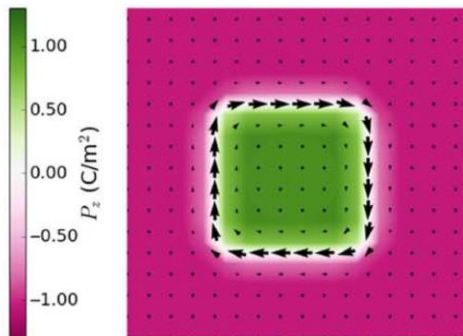
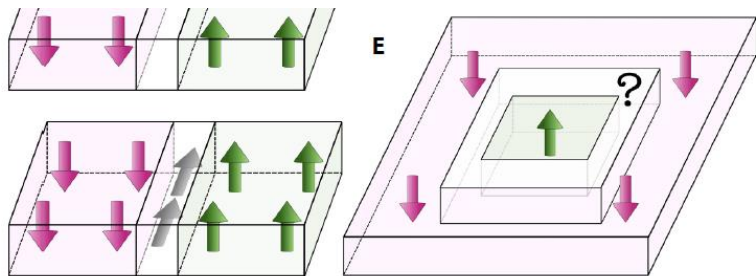
## 'Supercrystals'



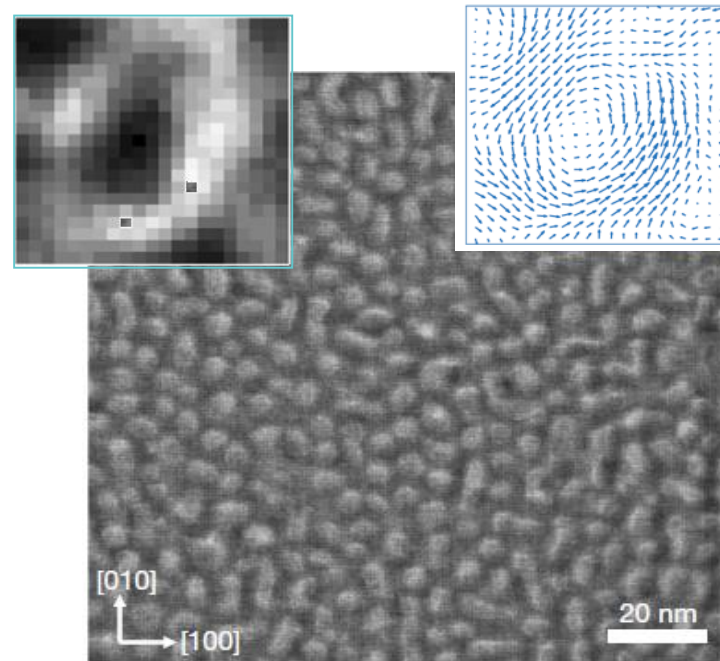
Hadjimichael et al. Nat. Mat. (2021)



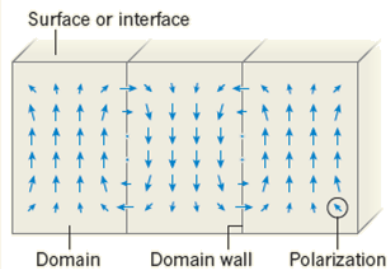




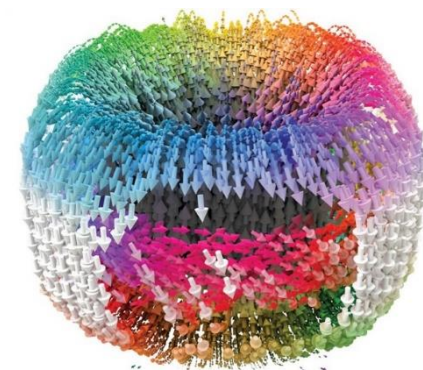
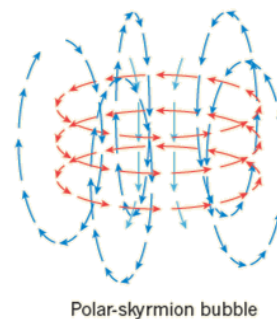
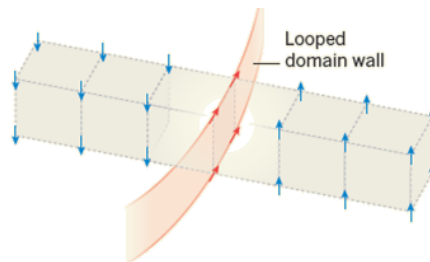
Pereira Goncalves et al. Science Advances 2019



Das et al. Nature 2019



Zubko, Nature N&V (2019)

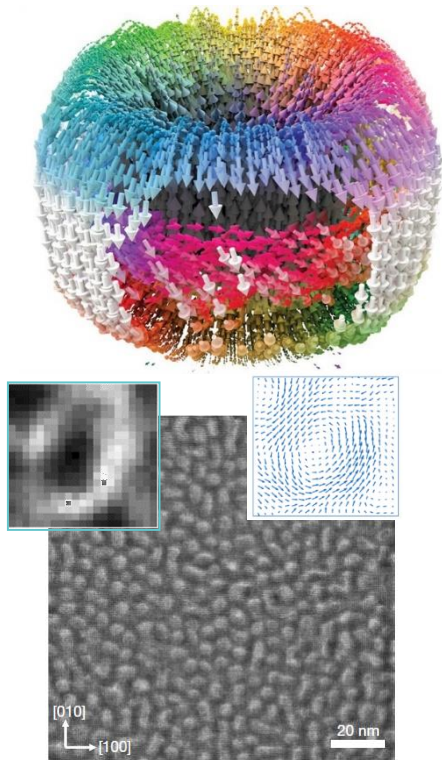
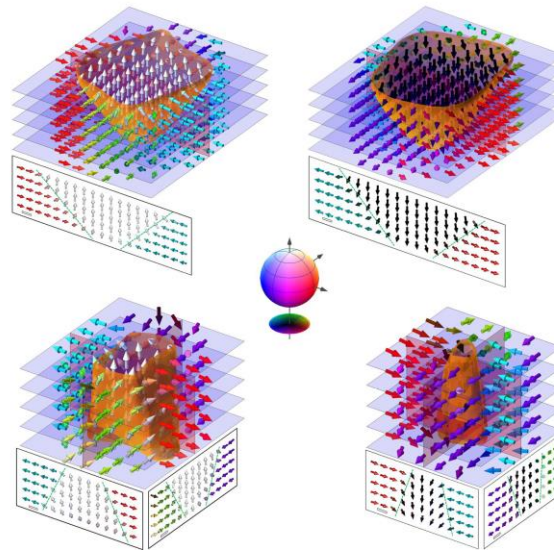


See lecture by Jorge Iñiguez



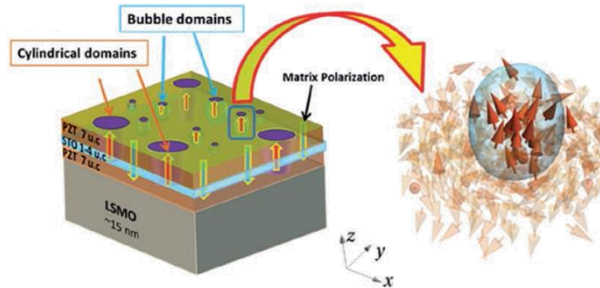
## Merons

Wang et al. Nat Mat 2020



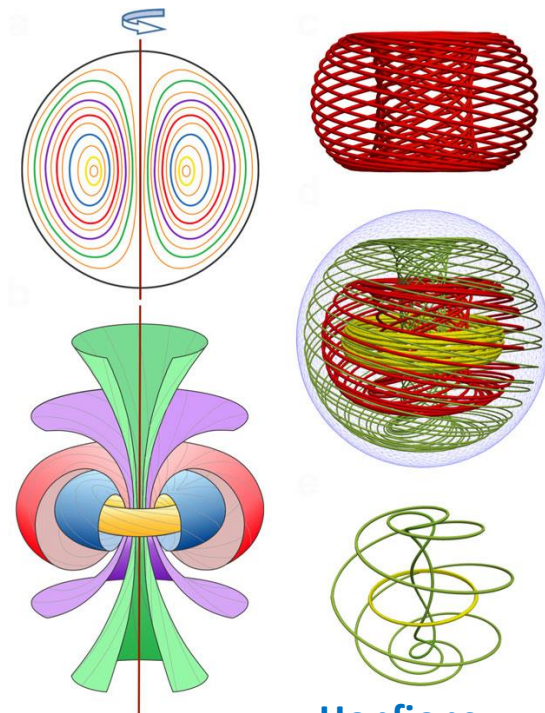
## Skyrmion bubbles

Das et al. Nature 2019



## Polar bubbles

Zhang et al. Adv Mat 2017  
Lichtensteiger et al. Nano Lett 2014



## Hopfions

Luk'yanchuk et al. Nat Comms 2020

See lecture by Jorge Iñiguez



- Hafnia ferroelectrics – [talk by Beatriz Noheda](#)
- Antiferroelectrics – [talk by Karin Rabe](#)
- 2D van der Waals ferroelectrics
- Domain wall functionality
- Strain engineering
- Flexoelectricity – [talk by Tae Won Noh](#)
- Ferroelectric photovoltaics
- Negative capacitance – [talk on Friday](#)
- Ferroelectric field effect devices & tunnel junctions
- Ferroelectrics for neuromorphic computing – [talk by Beatriz Noheda](#)
- Electrocalorics – [talk by Emmanuel Defay](#)
- Free-standing films – [talk by Mariona Coll](#)
- Multiferroics – [talk by Morgan Trassin](#)
- Topology in ferroelectrics – [talk by Jorge Iñiguez](#)
- Relaxors – [talk by Brahim Dkhil](#)
- Ferroelectric quantum criticality
- Artificially layered ferroelectrics
- ‘Ferroelectric metals’